PERT TIME/ COST

**an aid
to agribusiness
management**

Cooperative Extension

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PERT Time/Cost; An Aid to Agribusiness Management

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In 1958, a new managerial planning and control technique was developed. It was called PERT (Program Evaluation and Review Technique) and designed to provide: a) management information on actual and impending problems in completing a project; b) a continuous status report on active projects vis-**B**vis achieving established objectives, completion dates, and the probability of reaching both; and c) notation of most and least critical component activities within each project.

PERT presented a comprehensive illustration of all major project activities and their interdependencies. In fact, it even provided time requirements (hence called PERT Time) needed for completing each component activity. It focused managerial attention on those business activities most vital in meeting the project completion date and identified which resources could be used more effectively if transferred to other phases of a project. Finally, PERT provided a scheme of the project as it occurred, thereby illustrating the effects of managerial changes on the entire project.

The ability of PERT to predict future performance and potential future problems through frequency reporting marked its major departure from previous planning and control techniques which relied heavily on historical data.

On June 1,1962, Secretary of Defense Robert S. McNamara and Robert C. Seamans, Jr., Associate Director of N.A.S.A. adopted a system titled PERT Cost (i.e., the addition of $ cost dimension to PERT Time) as the standard supervisory format over major weapons and space programs. Hence, a second useful dimension was added to the already time-tested PERT Time system.

Objectives

By the mid-1960's PERT planning and control systems were causing a great stir throughout this nation's managerial circles. The more progressive businesses were rapidly adopting PERT, making modifications and improvements. Some industries looked to PERT as the cure-all for management problems. A few industries chose to observe (cautiously) others' use of PERT and some even ignored it entirely. The agribusiness industry fell into this category. Extremely few agricultural businesses adopted PERT and a majority of agribusiness managers knew nothing of its existence.

PERT has existed for almost 10 years; and while it may never prove to be the panacea many had hoped, it would, in my opinion, behoove agribusiness managers to understand its principles and technique, and be able to use its information in decision making.

This manual gives agribusiness managers the knowledge needed to apply PERT effectively. It first describes PERT fundamentals and then introduces the reader to its basic rules and practices. The objective is not to make agribusiness managers into operations research technicians, but to improve their competence in project planning and control. To become more competent, the reader need not become a statistician. However, he will have to develop an intuitive understanding of the statistical fundamentals of the PERT system.

In short, the objectives of this manual are two-fold:

1)   To enable the reader to assemble the necessary information and construct a PERT network.

2)   To enable the reader to utilize a PERT network or PERT analysis sheet in project planning, control and decision making.

What Is PERT?

Definition: PERT is a management tool for defining and integrating events; a process which must be accomplished in time to assure completing project objectives on schedule.

Three basic factors influence project progress: a) time, b) resources, and c) technology. Under PERT Time, time becomes the common denominator of resource application and technological specifications. Resources, i.e., financial resources are incorporated into the basic system to form PERT Cost. Technology is considered by both PERT Time and PERT Cost as a "Facilities factor," i.e., technological requirements and/or changes can be incorporated into an established PERT network and the resulting implications readily observed.

PERT pinpoints, on one hand, the critical activities which may require remedial decisions, and, on the other, activities for which surplus time exists such that a delay would not prohibit project achievement on time. As a management tool, therefore, PERT assists the decision maker, but does not decide for him. It can be introduced in any phase of an on-going project; however, it is most effective when used at the outset of a project.

In short, the PERT system reflects the complex interrelationships of activities leading toward the end of the project.

Objectives: PERT's objectives are: To provide, through applying an integrated management information system (which contains a balanced combination of the basic elements of time, cost, and performance), coordinated planning and control information at the proper levels so that timely managerial judgments will meet all established project objectives.

Attributes: PERT's inherent planning technique forces the manager to develop a comprehensive plan, and allows for realistic scheduling. The procedure for monitoring, forecasting, and simulating allows the manager to respond quickly to unexpected changes in the project, detect trouble areas early, and evaluate proposed alternative courses of action. PERT also allows the

manager to evaluate alternatives at any time during a project's duration by seeing how each would affect its completion date. One benefit of PERT which no comparable technique provides is its capacity to forecast problem areas well in advance. Once he identifies the critical path, the manager can take immediate remedial action and, thereby, perhaps restore the project to its schedule. Finally, PERT will allow management to simulate the effect of alternative decisions and study their effect on project deadlines before the program has begun.

Procedure

This manual familiarizes the reader with the components, characteristics and uses of the PERT technique. The first section of the manual, discusses the basic PERT format, i.e., it describes the component parts, conceptual framework, functional organization, etc. Later the manual considers the actual construction of a PERT network and includes more complex operations. Finally, the manual describes the managers' use of PERT in typical planning and control areas. Throughout, the manual provides examples of terminology, techniques, and routine overall operation. In fact, PERT's schematic orientation should, in itself, help readership understanding. Unless otherwise noted, PERT will refer to PERT Time as opposed to PERT Cost. The concluding section of the manual covers the differences and attributes of each.

Introduction

PERT is a manager's tool for defining and coordinating moves for completing a project's objectives on time. Its use is not restricted to the business world. It can be applied to any endeavor which requires planned, controlled, and integrated work patterns.

More often than not, diversified activities contribute to the difficulty in completing a project on schedule. Many uncertainties are associated with these activities. PERT is a technique that statistically presents knowledge about these uncertainties.

To show this more clearly, let's view the preparation of this manual, as the project under consideration. Obviously, many different activities, such as writing, reviewing, editing, and printing, will contribute to preparing a PERT management manual. Many uncertainties are associated with these activities. In our discussion, we will use PERT to generate statistical information about the uncertainties. This information will then be used to establish better managerial control over the project.

Before discussing technical terminology, one last descriptive statement about PERT should be made. PERT is defined as:

1)   A manager's tool -- for defining and coordinating what must be done to accomplish project objectives on time.

2)   A technique -- that aids the decision-maker, but does not make decisions for him.

3)   A technique -- that presents statistical information regarding the uncertainties associated with completing the different activities inherent in the project.

4)   A method -- for focusing a manager's attention on: a) latent problems that require solutions, and b) procedures and adjustments of time, resources, or performance, which may improve the probability of meeting all intended project completion dates.

Technical Terminology

Event: An *event* as used in PERT, is the beginning or completion of a task, not its actual performance. For example, "write a management manual" is not an event, but "writing of a management manual completed" is. In other words, one must begin and complete certain significant activities before the manual project will be done. These beginning and ending points are titled events and used by the manager to keep track of project progress. Events, therefore, consume neither time nor resources, but are simply notations of significant points in the project. A PERT event must meet the following three criteria:

1)   It is the beginning or the completion of a task.

2)   It does not consume time or resources.

3)   It is a significant point in the project.

Now visualize the vast number of events that occur in any large project. For example, consider our project of preparing this manual. The number of events in this project will depend on the amount of detail that you, as the manager, require to supervise the project. Generally speaking, the more detail, the greater the required number of events, and the better your ability to maintain control.

Events may be represented in a PERT network by any selected geometric figure, e.g., ovals, circles, squares, etc. The events must, however, follow logically. For example, consider the following series of events and arrange them in proper sequence.

1)   Background research completed.

2)   Final manuscript typing finished.

3)   Manuscript review begun.

4)   Printing of manual begun.

5)   First draft of manual manuscript completed.

Obviously, the background research must precede the final manuscript typing. However, final manuscript typing would have to follow the manual's first draft and the beginning of the

manuscript review. Finally, the printing of the manual could not begin until all other events had occurred. Therefore, the logical sequence of these events is: 1, 5, 3, 2, 4. The sequence depicted as a PERT network, would appear as shown in Figure 1.

Figure 1

PERT Network Sequence
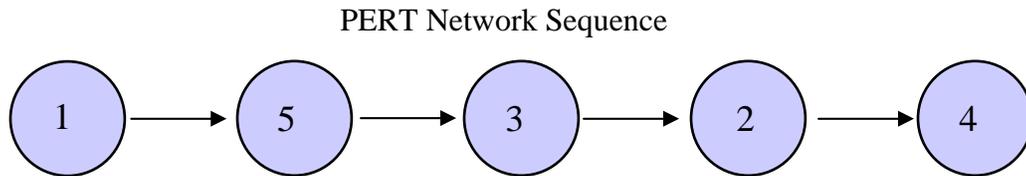
The arrows indicate the flow in the PERT network and the numbers identify the events. Notice the arrows, and not the numbers, indicate the order of events. Events that immediately follow one another are called successor events. Similarly, a predecessor event is one which immediately precedes another event.

Activity: The actual performance of a task is called an *activity*. Obviously, activities will occur between ALL events, i.e., a task begun cannot be completed until it has been performed. An activity, therefore, will always link two consecutive events in a PERT network and is diagrammatically represented by an arrow. Activities are the time- and resource-consuming portions of a PERT network. An activity is represented in a PERT network by an arrow which links two or more successive events and denoted by letters as shown in Figure 2.

Figure 2

PERT Network Activities
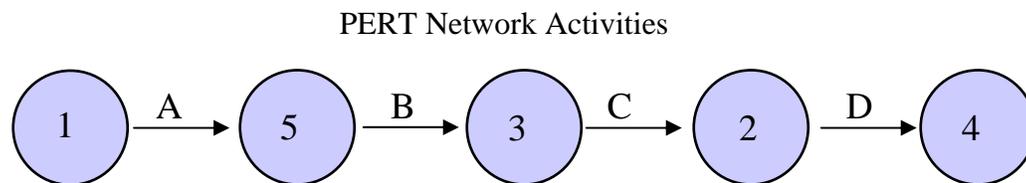
Network: The PERT network is a diagram of a managerial plan showing the steps needed to reach a stated objective. It depicts events and activities and the interrelationship, and recognizes the progress that must be made in one activity before subsequent activities can begin. The network must be comprehensive and include all important interdependencies and interactions. In other words, a PERT network is a flow chart composed of interdependent events and activities, each of which must be completed before the project objective is achieved. A network is shown in Figure 3.

Figure 3

PERT Network
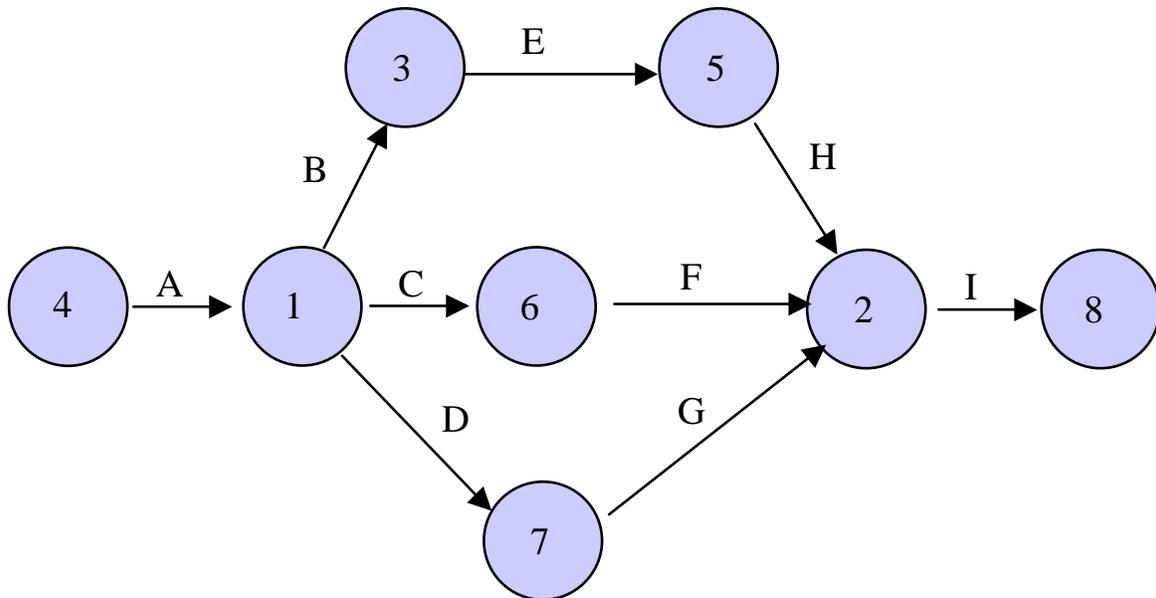
In our illustration, event 1 is a predecessor to events 3, 6, and 7. Event 2 is a successor to events 5, 6, and 7. All events are interdependent and linked by activities A-I. All activities and events must be reached and accomplished in sequence before the project is complete, i.e., event 1 cannot be reached until activity A has been accomplished, event 5 cannot be reached until activity E has been accomplished, etc. These relationships are often referred to as constraints.

Some key features of a PERT network are:

1)   Events must take place in a logical order.

2)   Activities represent the time and the work it takes to get from one event to another.

3)   No event can be considered reached until ALL activities leading to the event are completed.

4)   No activity may be completed until the event preceding it has been reached.

Network Construction

Because the network is the basis for the entire PERT system, its construction is best treated separately. This construction is vital to your understanding of PERT.

To accomplish a project's objectives successfully requires a managerial plan that identifies specific activities which must occur in sequence because of their interdependencies. Before he attempts to develop a network, it is often helpful for the manager to assemble a planning team

composed of representatives of each function to be performed. If our project objective were to prepare this management manual which you are now reading, the planning team might consist of: a) myself, as manager (author) and team leader, b) an editor, c) an administrative representative, d) a typist, e) a printer, and f) a draftsman. To give the manager the opportunity for improved planning and greater control, each team member would be asked to break his specific function (as it applies to the project) down into a sequence of activities. Before constructing the network, the manager should prepare a list of the teams' total activities. During his review of this list, he will probably discover many interface-type activities of which he was previously unaware, i.e., they may have been "understood" but not formally recognized nor specified and assigned. The editor, for example, may explain that his sequence of activities is dependent, in part, on the draftsman's ability to supply drawings of specified size-reductions on a rather rigid time schedule. Or, the administrative representative may point out that one of his activities (conducting a formal departmental review) is time-static because of a departmental ruling requiring that each reviewer be given a minimum of three weeks to complete his task.

Also, when the manager actually begins to draw the network, more activities than those originally envisioned by the planning team will likely appear. He will discover, for example, that a two-day lag exists between printing and distribution activities because the printed manuals must be moved from the print shop to the mail room. Of course, this is one of PERT's strengths; a properly constructed network shows ALL activities necessary to reach the project objective and provides a convenient check-off procedure to insure its success.

Networks are considered working papers, i.e., no draftsman or commercial artist need be employed to construct them. The only requirement is that the network information be readable. For example, constructing a network according to a time scale (e.g., using one inch equal one week) rarely proves worthwhile. Completion times are often changed during the project. Sometimes the entire plan is altered. This highlights another PERT attribute; it is dynamic, changing as the need arises yet remaining valid and useful. In fact, a PERT network often changes with each analysis, particularly early in a project.

Once the network has been constructed and approved by the planning team, it becomes a "master plan"; i.e., the schedule should be firm, and rigid rules for altering it should be established as standard practice to insure its reliability. Always remember, the network may have been constructed according to a contract commitment on the project completion date. Any change in it may, therefore, require a change in the contract.

Construction Planning: In planning the construction of a PERT network, certain "rules-of-the-road" must be followed:

1)   The activities do not represent alternate routes -- each road must be traversed to reach the network destination.

2)   The arrival at a road end cannot occur until each preceding road transversal has been completed.

3)   Travel succeeding an arrival cannot be started until the arrival has occurred.

4)   Arrival at a particular location cannot occur twice, i.e., the network loop may not show an encircling flow.

5)   Only one road transversal may join any pair of arrivals.

6)   Every network route plan can have only one original departure and one final arrival.

Let us now use our definitions of activities, events, their diagrams and our rules-of-the-road to construct some simple networks.

More than one activity can emanate from an event or go into an event. When two or more activities go into an event, the event is called a node point, see Figure 4.
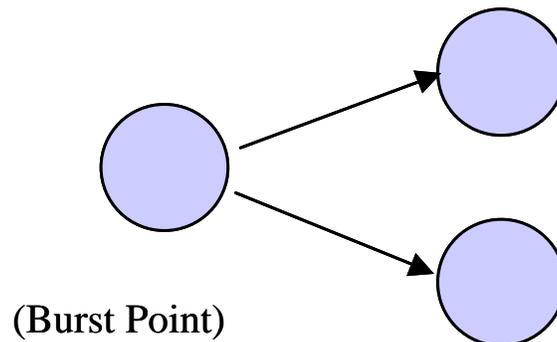
Figure 4

Node Point

(Node Point)
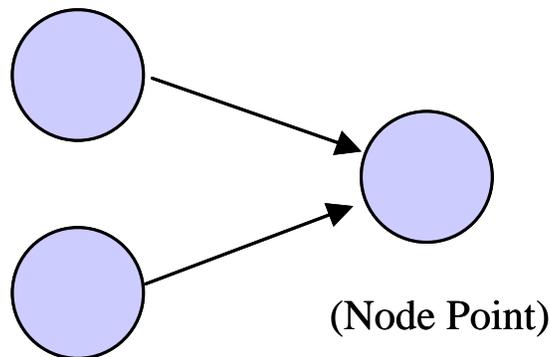
When two or more activities emanate from an event, the event is called a burst point, see Figure 5.

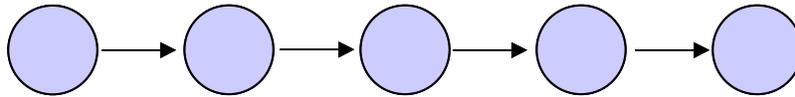Figure 5

Burst Point

(Burst Point)

When activities are related and follow one another in a series, the series is called a path, see Figure 6.
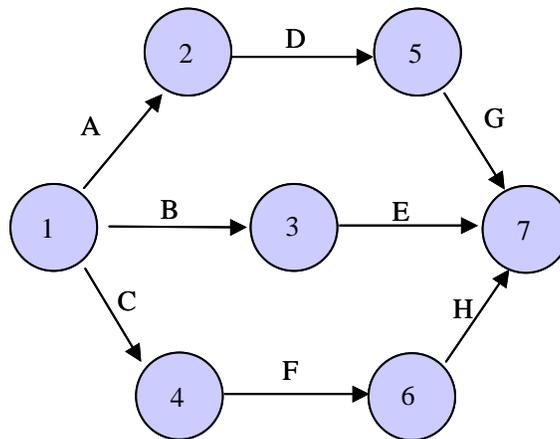
Figure 6

PERT Path

Now we shall construct a simple PERT network showing several paths to the end event, see Figure 7.

Figure 7

PERT Network

You will note that activities A-D-G, B-E, and C-F-H are on different paths, all emanating from the burst point 1 and into the node point 7.

Returning to our management manual project, let us consider the following list of activities, set them in a proper order, and construct the appropriate PERT network. (Note: not all activities of the complete project are shown.)

  (A)   Type rough draft of manuscript.
  (B)   Notify editing staff.
  (C)   Prepare manuscript outline.
  (D)   Receive editing's acknowledgement of funding.
  (E)   Develop idea of preparing management manual.
  (F)   Determine need for management training.
  (G)   Prepare manual project proposal.
  (H)   Receive notification of administrative approval.

(I)   Write manuscript rough draft.
(J)   Establish editing time schedule.
(K)   Conduct background and literature research on topic.
(L)   Request administrative approval of proposal.

Typing the manuscript rough draft does not represent the final project activity, of course, but it is the successor event to all those activities listed above. While it may be modified, the basic PERT network should have been constructed as shown in Figure 8.

Figure 8

PERT Project Network
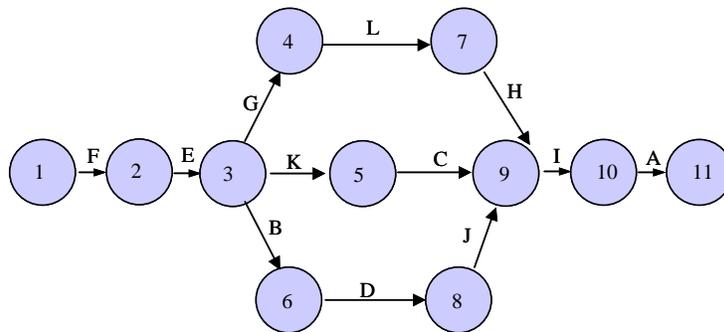
This is an activity-oriented network, i.e., one where activities are placed in time sequence, while events are numbered for convenience. Had events rather than activities been listed above (an event-oriented network), network construction would have differed only in so far as events would have been reorganized in sequence while activities were lettered for convenience. I believe the activity-oriented network is more widely used, easier to understand, and easier to update. Therefore, the remainder of this manual will deal only with activity-oriented PERT networks.

Let's return to our illustration and complete the construction of the project. The last activity plotted was typing the manuscript rough draft. Beginning at this point, we list the following activities:

(M)   Drafting manuscript diagrams.
(N)   Conducting a departmental informal review.
(O)   Conducting an Extension formal review.
(P)   Editing.
(Q)   Printing.
(R)   Typing the final edited manuscript.
(S)   Notifying the print shop.
(T)   Submitting drafted diagrams to print shop.
(U)   Submitting drafting requirements.
(V)   Retyping 2nd revision.

(W)   Rewriting 2nd revision.
(X)   Retyping 1st revision.
(Y)   Rewriting 1st revision.
(Z)   Establishing printing schedule.
(AA)   Submitting final manuscript to print shop.
(BB)   Preparing 2nd revision for editing.
(CC)   Preparing 1st revision for editing.
(DD)   Distributing printed manuals through the mail.
(EE)   Moving printed manuals from print shop to mail room.

Setting these activities into a logical sequence and completing construction of our project network, we would have the network shown in Figure 9.

Figure 9
PERT Project Network
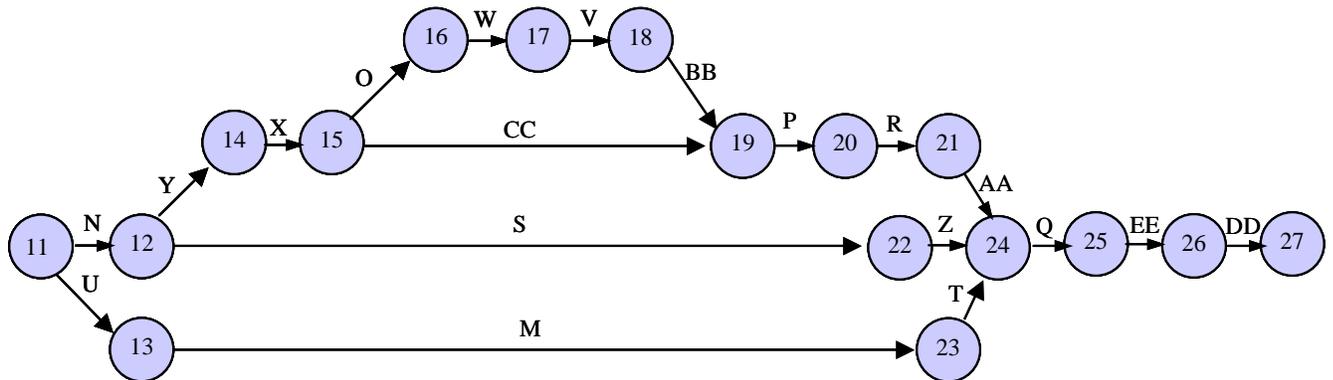
As now shown, event 1 would begin the project and event 27 would complete it, with 25 intervening events and 31 intervening activities.

Our PERT network is now constructed, with all events and activities noted and properly sequenced. However, it is "time-neutral"; i.e., the plan has no time factors attached to it. This process of attaching such time factors is often referred to as scheduling.

Scheduling

Scheduling may be defined as translating a project plan into a timetable with specific calendar dates governing the beginning and completion of all project efforts. Scheduling, properly performed and approved by responsible management, should:

1)   Alert all persons engaged in the project of the timeliness of their functions and responsibilities.

2) Assist the manager to allocate and authorize the use of productive resources efficiently and to assure project completion on time.

3) Permit continuous comparable analysis of scheduled plan versus actual accomplishment, thereby enabling the manager to measure and evaluate the current project status.

Scheduling is a way of realistically timing a project and its inherent activities. It calls for skill and knowledge of the time and resource requirements of all activities and their relevant capacities and efficiencies. Scheduling is the manager's way of determining the resources required by several (perhaps competing) projects. The manager's responsibility is to balance these multiple (and sometimes competing) requirements for resources within realistic activity and project time constraints.

He translates an approved PERT network into a schedule by assigning the resources needed to complete the planned activities on time. The manager faces a major constraint in scheduling: he must conform to the network. He must maintain the interdependence between network construction and scheduling throughout the life of the project. He must avoid any tendency to disrupt the logical relationships between these two functions.

Principles involved in these vital relationships include the following:

1) The approved network must govern the sequence and content of the work to be performed.

2) Scheduling must validate the network by converting it to an acceptable timetable. If scheduling restrictions make it unable to validate the network, then the manager may adjust the network.

3) Scheduling may not change the planned sequence of activities and events. It will, however, with managerial approval, establish a timetable for governing the beginning and completion of activities and their resource expenditures.

4) Scheduling and network construction must be so performed that they are continuously related and, thereby, constitute only one plan for a given project, as viewed by the manager.

Once the network has been constructed, the next step is to develop a schedule to support it. Up to this point, time estimates have been ignored. Scheduling, however, requires the expected time for completing each network activity.

The same planning team originally selected by the manager is now asked to provide its best time estimates for each activity. Since they are functional experts, we expect team members to be relatively accurate. Furthermore, we expect each to commit his employees to the expected times for activities. In some cases, however, it may be necessary for him first to clear these commitments with other supervisors.

After the manager has received all inputs of expected times, the network is processed using the only dates known at that time, i.e., the project's beginning and completion dates (if contracted). The results will then be analyzed to determine if the time estimates submitted are realistic and will support the network under its contractual time obligations.

Estimates of Activity Times

Activity time is the elapsed time required for an activity. Estimating activity times is probably one of PERT's most critical features. Agribusiness firms are often so closely linked with agricultural production that they become as susceptible to seasonal fluctuations and market variations as producers. Consequently, agribusiness managers are reluctant to commit themselves to a rigid time schedule. Weather conditions, alone, prompt uncertainties and make it difficult for the manager to develop a single time estimate. However, experience has shown that managers are less reluctant if allowed three different estimates, especially when they understand PERT and how the concept of three time estimates is used. PERT, therefore, calls for not one, but three estimates of every activity time and allows the manager an opportunity to express his uncertainty about the possible time range of an activity.

All three time estimates assume a static level of resource use. The estimates should be as good as possible because PERT results depend directly on them. To obtain accurate estimates is not easy. It will require research, collaboration with planning team members, and homework. Simple guesswork is inadequate. If some time estimates are mere guesses, the manager will soon realize that they jeopardize or needlessly extend the project schedule date. Once the estimator realizes that his contributions are a small, but vital component of the PERT system, he will try to steadily improve his estimates. In short, guesswork will not replace intelligently derived estimates.

The person most familiar with the operation and requirements of each activity should submit the three time estimates. These should meet the following criteria:

    1)   *Optimistic Time --*   the minimum time period in which the activity can be accomplished, i.e., the time it would take to complete it if everything proceeded better than expected. (labeled a.)

    2)   *Most Likely Time --*   the best estimate of the time period in which the activity can be accomplished, i.e., the estimate submitted if one (only) had been requested. (labeled m.)

    3)   *Pessimistic Time --*   the maximum time period it would take to accomplish the activity, i.e., the time required if everything went wrong, excluding major catastrophes. (labeled b.)

It is acceptable to state these estimates in days, weeks, or months as long as the measure is used consistently. Once made, activity time estimates are firm and should not be changed without a change in the nature and scope of the activity or in the level of resources allocated to it. The following time relationships must be adhered to: $a. \leq m. \leq b.$

Time Distribution: When an estimator makes three time estimates for performing an activity, he implies the existence of a distribution of possible activity times which may approximate that shown in Figure 10, where a is the optimistic time, m the most likely time, b the pessimistic time, and t_e for each distribution, as shown below, the statistical mean or average value of the three time estimates. More simply, t_e is defined as the average time an activity would require were it repeated many times.

Figure 10

Frequency

Distribution of Possible

Activity Times
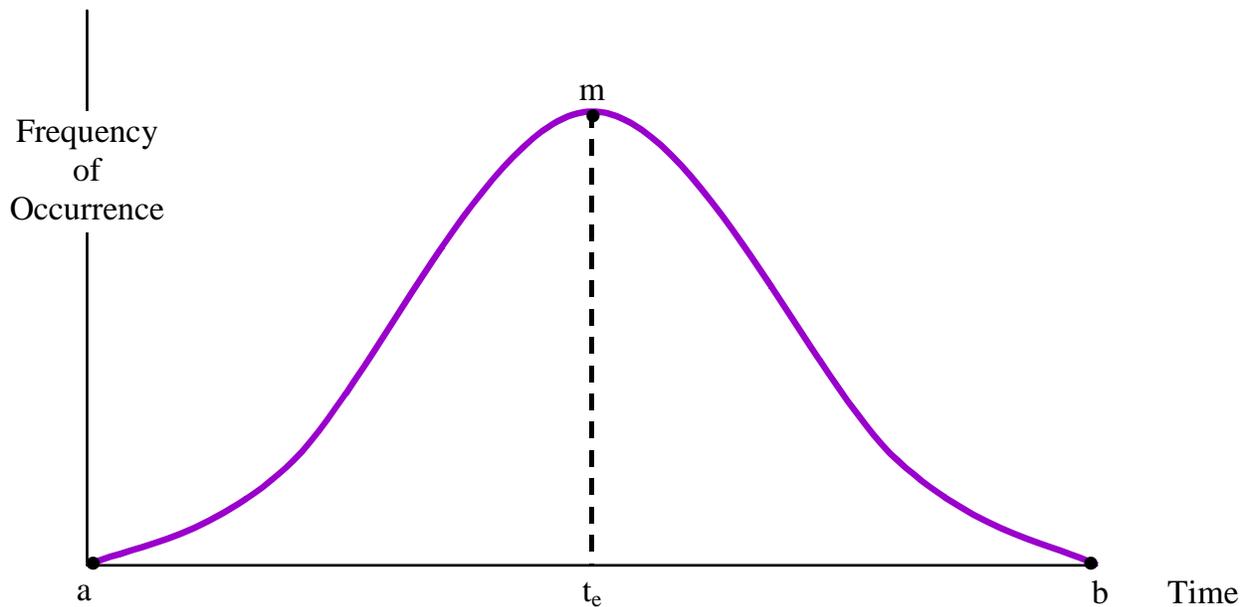
The relative values (positions) of a, m, and b on each distribution, of course, depends on the estimator's judgment and calculations. Once established, their relative positions on the distribution affect the value or the position of t_e. In fact, once the numerical values of a, m, and b have been determined, an estimate of t_e is relatively easy to determine. For use in the PERT procedure, an estimate of t_e is $\dfrac{a+4m+b}{6}$, and forms our basis for estimating the *expected value* of the activity performance time.

To further illustrate, let's assume that a planning team member is asked to provide time estimates (a, m, and b) for performing three different activities (A, B, and C) which normally fall within his functional area. The curves below represent his best estimates, see Figure 11.

Figure 11

Distribution of Activity Times
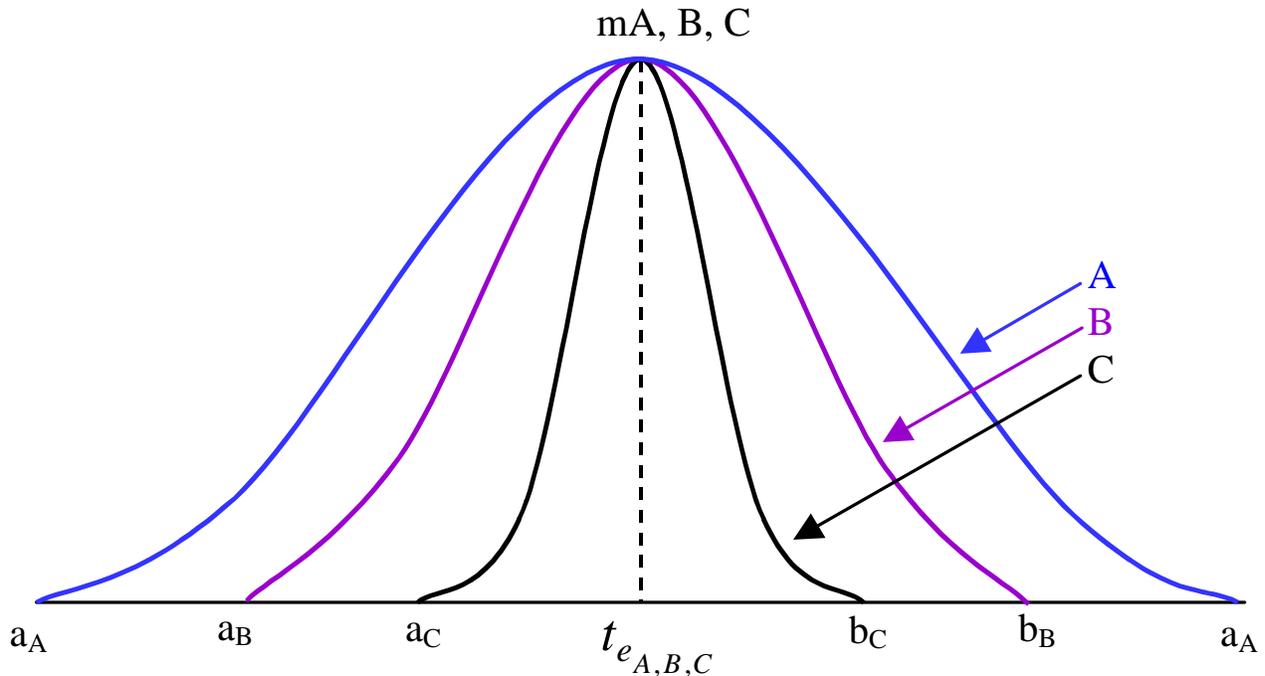
You will note that t_e of each curve is the same, as is m, but that the curves differ greatly in shape. The wider range of curve A reveals that the estimator was not able to predict the activity completion with accuracy (relative to B and C). But a paradox of sorts may exist here. The estimator's uncertainty may have resulted from either too much or too little information. If too little, the estimator may have given himself leeway via a wide range of estimates. If too much, the estimator might be told of uncontrollable factors affecting completion time, for which he must allow. Similarly, curves B and C could result from over-optimism based on limited information. Nevertheless, the more information one has about an activity, the tighter his time estimate distribution curves will become. Therefore, we can assume that had the estimator had more information, curve A might more closely have resembled curve C.

Variance: We may conclude that the wider the separation of optimistic and pessimistic estimates (i.e., the flatter the distribution curve), the greater the uncertainty attached to the activity time in question. To help us measure the degree of uncertainty we turn to a term called *variance*. The variance is a descriptive measure of the uncertainty associated with an activity time distribution; i.e., a large variance indicates great uncertainty, a small variance indicates a more accurate estimate. The symbol commonly used to denote variance is σ^2 and its value is compiled as follows:

$$\sigma^2 = \left(\frac{b-a}{6} \right)^2$$

How does the variance estimate work? Let's assume that your agribusiness firm is having an annual audit performed on a contract basis. Accountant X and accountant Y submit identical bids, i.e., identical service, work quality, and cost. However, when asked how long it would take to complete the audit, the two accountants responded as follows, see Table 1.

Table 1
Estimated Duration of Audit

	Optimistic Time (days)	Most Likely Time (days)	Pessimistic Time (days)
Accountant X	3	4	5
Accountant Y	1	4	7

On the average, both accountants would complete the audit in four days, i.e., expected activity time $t_{e_X} = \dfrac{3+(4)4+5}{6} = 4$, $t_{e_Y} = \dfrac{1+(4)4+7}{6} = 4$.

Which of the two seemed most certain about the expected duration of the audit? The variance of their estimates will help you answer this question. Variances are $\sigma_X^2 = \left(\dfrac{5-3}{6}\right)^2 = \dfrac{1}{9}$, $\sigma_Y^2 = \left(\dfrac{7-1}{6}\right)^2 = 1$. Since accountant Y's variance exceeds accountant X's, we may assume Y is less certain and accept the contract with X. Admittedly this illustration is oversimplified, but it shows how variance is used to measure an activity's uncertainty.

Let's return again to our management manual project. Based on my knowledge, and that of others involved in preparing such a manual, the following estimates of activity times have been made, see Table 2.

Table 2
Activity Time Estimates

Activity	Activity Times (days)				
	a	m	b	t_e	σ^2
F	8	10	15	10.500	1.360
E	8	10	12	10.000	.444
G	11	12	13	12.000	.111
K	18	20	28	21.000	2.776
B	1	2	3	2.000	.111
L	1	2	4	2.166	.250
D	1	4	8	4.166	1.360
H	1	2	3	2.000	.111
C	10	15	20	15.0	2.776
J	3	5	7	5.0	.444
I	30	40	55	40.833	17.356
A	5	8	12	8.166	1.360

So far, we have defined a, m, and b, illustrated how to calculate t_e and σ^2, and shown how all these items are interpreted and used. The next step is to establish the expected times for reaching network events. These are represented by the symbol T_E and appear above the events on the network. Before discussing this in detail, let's return to our example network and refresh our understanding of predecessor and successor events, see Figure 8, Table 3.

Table 3
Predecessor and Successor Events

Activity	Predecessor Events	Successor Events
F	1	2-11
E	1-2	3-11
G	1-3	4, 7, 9-11
K	1-3	5, 9-11
B	1-3	6, 8-11
L	1-4	7, 9-11
D	1-3, 6	8-11
H	1-4, 7	9-11
C	1-3, 5	9-11
J	1-3, 6, 8	9-11
I	1-9	10-11
A	1-10	11

The T_E of an event represents its *earliest possible completion time.* The T_E is computed for each event by adding the t_e's of the activity paths leading to the event.

In our example below, t_e's are placed above their respective activities and the T_E's above their respective events, see Figure 12.

Figure 12
Activity Completion Times
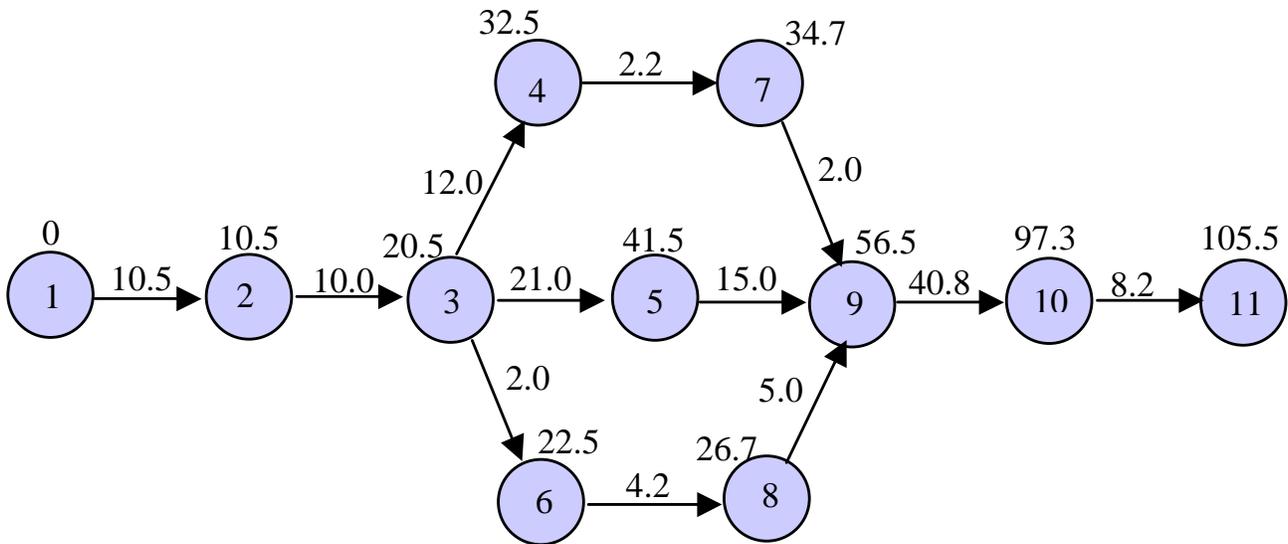
The time you can expect to reach event 1 is zero. No activity precedes the event; and therefore, no time can be consumed in reaching it; i.e., the T_E for event 1 is zero.

According to our network, activities F, E, G, B, K, L, D, H, C, J, I, and A must be completed before event 11 is reached. Since three paths lead to event 11, the longest time consuming path represents the earliest possible time you can expect to reach 11, i.e., 105.5 days via activities F, E, K, C, I, and A.

Slack: The next step is to determine the *latest allowable completion times* for each event. T_L is the symbol we shall use to denote this time. T_L times are derived from a previously established completion time or *contractual obligation date* which we shall refer to as T_S.

Returning to our example, let's assume that beginning with event 1, I had committed myself to reaching event 11 within 105.8 days. Our completion date for reaching it, therefore, is a T_S of 105.8 (working) days. The latest allowable completion time T_L for it is also 105.8 days. The latest allowable completion time T_L, therefore, coincides with the committed completion schedule T_S.

The values of T_L are computed for each event and placed below the events on the PERT network. The T_L's are calculated in a manner opposite to that used to determine the T_E's. You

begin with the last event and work back toward the first. To compute each event's T_L, you must subtract the value of the t_e from the successor event's T_L value. If more than one value of T_L is obtained (as in our example below), the smallest is selected. Our example would now appear as shown in Figure 13.

Figure 13
Activity Completion Times
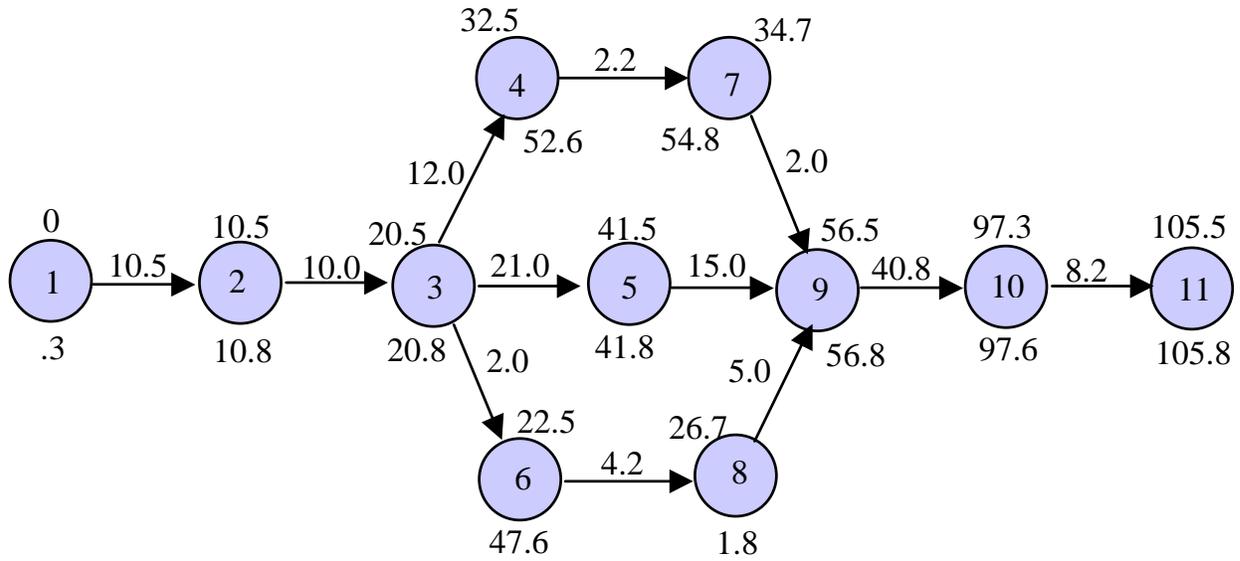
We have now computed the latest allowable time T_L and expected time T_E for each event. From these two values, we can now calculate the *slack* of an event, i.e., the slack of an event is $T_I - T_E$. If we are allowed 105.8 days to complete our project and can, if all goes as expected, perform it in 105.5 days, we have a .3 day grace period during which a delay can occur and still allow for completing the project on schedule. The value of slack can be either positive, negative, or zero, depending on the $T_L : T_E$ relationship. Looking at our network above we can compute the slacks shown in Table 4.

Which events in our example network indicate a possible application of excess resources? Events 6 and 8, and perhaps 4 and 7 show large slacks which may be indicative of excess resource (funds, material, and/or labor) application. It might be possible to switch resources from activities G, L, H, B, D, and J to activities K and C, thereby reducing the slack on events 4, 6, 7, and 8, and permitting earlier completion of events 5, 9, 10, and 11. Calculating each event's slack also helps the manager spot potential trouble areas, i.e., those activities in which any delay exceeding .3 day would delay project completion.

Table 4
Slack Computations

Event	T_L	T_E	Slack
1	.3	.0	.3
2	10.8	10.5	.3
3	20.8	20.5	.3
4	52.6	32.5	20.1
5	41.8	41.5	.3
6	47.8	22.5	25.1
7	54.8	34.7	20.1
8	51.8	26.7	25.1
9	56.8	56.5	.3
10	97.6	97.3	.3
11	105.8	105.5	.3

Of the three types of slack:

1)   Positive slack indicates ahead-of-schedule conditions where excess resource application may be occurring.

2)   Zero slack indicates an on-schedule condition with adequate resource application but no margin of safety.

3)   Negative slack indicates a lagging schedule and implies insufficient resource application.

Critical Path

It is important to notice that the slack value associated with each event indicates how critical it may become to that project schedule. A small slack indicates only minor slippage in activity times can be permitted. The less its slack, the more critical the event, (particularly if the slack shows signs of lessening).

In any project, several paths may lead from the initial to the terminal event. One however will be more critical than the others and is called the *critical path*. In our example, the path through event 5 is critical because it allows a slippage of only .3 day.

The critical path has two characteristics which make it even more important in project analysis.

1)   It requires the most time to get from the initial event to the final event.

2)   Any event on it that slips in time will cause the final event to slip by the same amount in which case the slippage exceeds the slack.

Probability Factors

So far, we have used the symbol T_S to represent the scheduled project time. However, it seems likely that events other than the terminal event may be important enough to be assigned a completion date. If such strategic events do appear within a network, there is no reason why one could not associate a T_S with them.

It would aid agribusiness managers if some way existed for estimating the probability of reaching these T_S values. For this aid, we again turn to a relatively simple two-step statistical technique for computing a *probability factor*, i.e., the probability of meeting a scheduled date. Let's now discuss the first step.

Z Values: In our first step, the probability factor is computed by the following formula:

$$Z = \frac{T_S - T_E}{\sqrt{\sum \sigma_{T_E}^2}}$$

To get the numerator, you simply subtract the expected time T_E for the event from the scheduled date T_S. For the denominator you add all the variances of the same activities used to calculate the T_E for the event, and take the square root of this sum.

Our example in Table 5 shows the calculated variances.

Table 5 - Activity Variances

Activity	Predecessor Event	Successor Event	a	m	b	t_e	σ^2
F	1	2	8	10	15	10.5	1.36
E	2	3	8	10	12	10.0	.44
G	3	4	11	12	13	12.0	.11
K	3	5	18	20	28	21.0	2.78
B	3	6	1	2	3	2.0	.11
L	4	7	1	2	4	2.2	.25
D	6	8	1	4	8	4.2	1.36
H	7	9	1	2	3	2.0	.111
C	5	9	10	15	20	15.0	2.78
J	8	9	3	5	7	5.0	.44
I	9	10	30	40	55	40.8	17.36
A	10	11	5	8	12	8.2	1.36

Let's assume that event 9 is important to the project, and we wish to determine the probability of our reaching it on schedule.

First we must set the schedule. As we saw earlier, the expected time for an event, T_E is computed by adding the t_e's of the activity paths leading to it and selecting the greatest. According to our network, activities F, E, G, K, B, L, D, H, C, and J must be completed before event 9 is reached. Since three paths lead to event 9, the sum of t_e's over the longest path will determine our T_E, i.e., in this case $T_E = 56.5$ days. Suppose we schedule a 56.8 day period to reach event 9, i.e., $T_S = 56.8$. The longest path for reaching event 11 and computing T_E is via activities F, E, K, and C. The sum of variances for these activities is
$$1.36 + .44 + 2.78 + 2.78 = 7.36 = \sum \sigma_{T_E}^2 .$$

Our formula for determining the probability of reaching event 9 on or before the schedule date is:
$$Z = \frac{56.8 - 56.5}{\sqrt{7.36}} = \frac{.3}{2.718} = .110$$

Notice that Z can be either positive or negative, depending on the relationship of T_S to T_E. Now that Z is determined, we move to step number 2.

P_R values: In the second step, the task is to convert the Z value into a probability value $\left(P_R\right)$.

From our example, we find a Z value of .110. Using Table 6, we can convert this Z value of .110 into a P_R value of .5438 by simply interpolating between values of .2 = .5793 and .1 = .5398. The probability of reaching event 9 in 56.8 working days or less is 54.38%. In other words, your odds are only slightly better than 50-50 that event 9 will be reached on schedule. This probability varies in direct proportion to the amount of time that T_S exceeds T_E and in indirect proportion to the variances of activity times. The logical inferences behind these relationships should be apparent.

Network Review
To review what we have covered, let's return to our management manual project. If you recall, the activities were as follows:

(A)　Type rough draft of manuscript.
(B)　Notify editing staff.
(C)　Prepare manuscript outline.
(D)　Receive editings acknowledgement of funding.
(E)　Develop idea of preparing management manual.
(F)　Determine the need for management training.
(G)　Prepare manual project proposal.
(H)　Receive notice of administration approval receipt.
(I)　Write manuscript rough draft.
(J)　Establish editing time schedule.
(K)　Conduct background and literature research on topic.

(L)   Request administrative approval of proposal.
(M)   Draft manuscript diagrams.
(N)   Conduct departmental informal review.
(O)   Conduct Extension formal review.
(P)   Edit.
(Q)   Print.
(R)   Type final edited manuscript.
(S)   Notify print shop.
(T)   Submit drafted diagrams to print shop.
(U)   Submit drafting requirements.
(V)   Retype 2nd revision.
(W)   Rewrite 2nd revision.
(X)   Retype last revision.
(Y)   Rewrite 1st revision.
(Z)   Establish printing schedule.
(AA)   Submit final manuscript to print shop.
(BB)   Prepare 2nd revision for editing.
(CC)   Prepare 1st revision for editing.
(DD)   (DD)Distribute printed manuals through the mail.
(EE)   Move printed manuals from print shop to mail room.

Table 6
Table of Values of the Standard Normal Distribution Function*

Z	P_R	Z	P_R	Z	P_R	Z	P_R
-3	.0013	-1.5	.0668	.0	.5000	1.5	.9332
-2.9	.0019	-1.4	.0808	.1	.5398	1.6	.9452
-2.8	.0026	-1.3	.0968	.2	.5793	1.7	.9554
-2.7	.0035	-1.2	.1151	.3	.6179	1.8	.9641
-2.6	.0047	-1.1	.1357	.4	.6554	1.9	.9713
-2.5	.0062	-1.0	.1587	.5	.6915	2.0	.9772
-2.4	.0082	- .9	.1841	.6	.7257	2.1	.9821
-2.3	.0107	- .8	.2119	.7	.7580	2.2	.9861
-2.2	.0139	- .7	.2420	.8	.7881	2.3	.9893
-2.1	.0179	- .6	.2743	.9	.8159	2.4	.9918
-2.0	.0228	- .5	.3085	1.0	.8413	2.5	.9938
-1.9	.0287	- .4	.3446	1.1	.8643	2.6	.9953
-1.8	.0359	- .3	.3821	1.2	.8849	2.7	.9965
-1.7	.0446	- .2	.4207	1.3	.9032	2.8	.9974
-1.6	.0548	- .1	.4606	1.4	.9192	2.9	.9981

* This common table can be found in most mathematics and statistics handbooks.
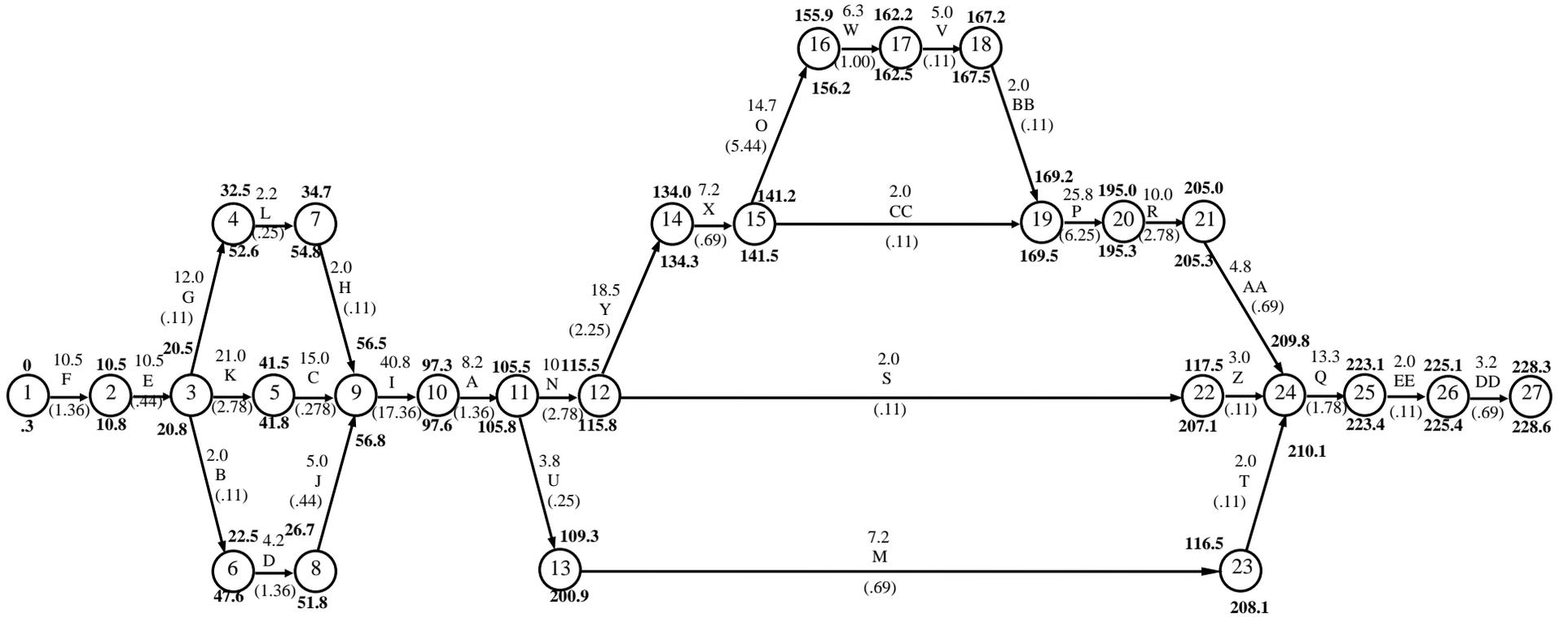
Figure 14
Agribusiness Management Manual
PERT Network

Table 7 - PERT
Activity Output – Slack Sort

Activity	Predecessor Event	Successor Event	Actual Date	a	m	b	t_e	σ^2	T_E	T_L	T_S	Slack	T_S-T_E	$\sum \sigma_{T_E}^2$	Z	P_R(%)
Project Initiation	(1)								(0)	(.3)		(.3)				
F	1	2		8	10	15	10.5	1.36	10.5	10.8		0.3				
E	2	3		8	10	12	10.0	0.44	20.5	20.8		0.3				
G	3	4		11	12	13	12.0	0.11	32.5	52.6		20.1				
K	3	5		18	20	28	21.0	2.78	41.5	41.6		0.3				
B	3	6		1	2	3	2.0	0.11	22.5	47.6		25.1				
L	4	7		1	2	4	2.2	0.25	34.7	54.8		20.1				
D	6	8		1	4	8	4.2	1.36	26.7	51.8		25.1				
H	7	9		1	2	3	2.0	0.11	56.5	56.8		0.3				
C	5	9		10	15	20	15.0	2.78	56.5	56.8	56.8	0.3	0.3	7.36	0.110	54.38
J	8	9		3	5	7	5.0	0.44	56.5	56.8		0.3				
I	9	10		30	40	55	40.8	17.36	97.3	97.6		0.3				
A	10	11		5	8	12	8.2	1.36	105.5	105.8	105.8	0.3	0.3	26.08	0.059	50.23
N	11	12		5	10	15	10.0	2.78	115.5	115.8		0.3				
U	11	13		2	4	5	3.8	0.25	105.3	200.9		91.6				
Y	12	14		15	18	24	18.5	2.25	134.0	134.3		0.3				
S	12	22		1	2	3	2.0	0.11	117.5	207.1		59.6				
M	13	23		5	7	10	7.2	0.69	116.5	208.1		91.6				
X	14	15		5	7	10	7.2	0.69	141.2	141.5		0.3				
O	15	16		7	15	21	14.7		155.9	456.2		0.3				
CC	15	19		1	2	3	2.0	0.11	169.2	169.2		0.3				
W	16	17		5	8	11	6.3	1.00	162.2	162.5		0.3				
V	17	18		4	5	6	5.0	0.11	167.2	167.5		0.3				
BB	18	19		1	2	3	2.0	0.11	169.2	169.5	169.5	0.3	0.3	38.46	0.048	50.19
P	19	20		20	25	35	25.8	6.25	195.0	195.3		0.3				
R	20	21		5	10	15	10.0	2.78	205.0	205.3		0.3				
AA	21	24		2	5	7	4.8	0.69	209.8	210.1		0.3				
Z	22	24		2	3	4	3.0	0.11	209.8	210.1		0.3				
T	23	24		1	2	3	2.0	0.11	209.8	219.1		0.3				
O	24	25		8	14	16	13.3	1.78	223.1	223.4		0.3				
EE	25	26		1	2	3	2.0	0.11	225.1	225.4		0.3				
DD	26	27		1	3	6	3.2	0.69	228.3	228.6	228.6	0.3	0.3	50.76	.042	50.17

a = optimistic time
m = most likely time
b = pessimistic time

t_e = average activity time $= \dfrac{a + 4m + b}{6}$

Slack = T_L – T_E for each event

σ^2 = activity time variance $= \left(\dfrac{b - a}{6}\right)^2$

T_E = earliest possible time that an event can be reached = $\sum t_e \text{'s}$ of activities on longest path to the event.

T_L = latest allowable completion time for event $= T_S - \sum_{t_e}$ of activities on shortest path to the event

T_S = scheduled completion time of an event

$Z = z - value - \dfrac{T_S - T_E}{\sqrt{\sum \sigma_{T_E}^2}}$

P_R = probability of event being reached on or before schedule.

These activities were then scheduled (i.e., listed by logical time sequence and separated by appropriate events) and a PERT network was constructed. This network appears in Figure 14.

As estimates of activity times became available (see Table 7), numerous computations were made; and the information obtained was added to the PERT network (Figure 14) as follows:

1) Activities appeared as arrows and were lettered A through EE.

2) Events appeared as circles and were numbered 1 through 27.

3) T_E's were computed and placed above each relevant event.

4) T_L's were computed and placed below each relevant event.

5) t_e's were calculated and placed above each activity letter.

6) σ^2 for each activity was calculated and placed below each activity letter and shown in parentheses.

The fully constructed PERT network now provides the manager with a wealth of information (in diagram form) for supervising and controlling the project.

For example, PERT has identified the critical path as being F, E, K, C, I, A, N, Y, X, O, W, V, BB, P, R, AA, Q, EE, and DD. The manager will watch these activities closely because he knows that a delay in completing any of over .3 day will set the entire project behind schedule. Other activities such as CC, S, and M are less critical. The manager knows that if all goes well, event 22 will be reached after 117.5 working days. However, he also knows that the project completion date won't be affected by a delay in activity S that places event 22 at 207.1 days. Finally, the manager is told that, whenever possible, he should switch resources from large slack activities to activities lying on the critical path.

PERT also alerts the manager to a crisis situation and allows for immediate adjustment. Suppose, for example, that at the time event 2 is reached, the manager receives word from the editing staff that all available funds have been frozen until the end of the fiscal year, 40 working days away. Now instead of 4.2 days, activity D (receive editing's acknowledgement of funding) will take 40 days, and it will not be possible to reach event 9 in 56.5 days as planned. Notice that the critical path has now changed from F, E, K, and C to F, E, B, D, and J. Several alternatives still exist for the manager. He can: a) reschedule the remaining activities and the final competition dates, b) try to convert excess resources from slack activities into the critical path to reestablish the original schedule later in the project, or c) he can combine a) and b) simultaneously. Whatever he decides, he can easily insert his adjustments into the network diagram and make it viable and useful.

PERT is a managerial technique which keeps the project on schedule and reduces its performance time. There are many ways to achieve these objectives.

One is to reduce the expected time for particular activities. Suppose in our example the scheduled completion date were reduced to 200 days, i.e., $T_S = 200$. As manager, you would look at two PERT paths for possible reductions; via 11, 12, 14, 15, 16, 17, 18, 19, 20, 21, and 24 which is the critical path with a large negative slack; and 11, 12, 14, 15, 19, 20, 21, and 24 which is semi-critical with a smaller negative slack. The two remaining paths show positive slacks and would allow for project completion in less than the required 200 days. Now let's assume that in discussing the problem with your employees and supervisors you discover that under special circumstances, activities O, W, V, and BB can be completed in only 1.2 days. The path of 11, 12, 14, 15, 19, 20, 21, and 24 is unaffected by this change and hence now becomes the critical path.

This emphasizes the importance of PERT in managerial analysis. The reduction of 26.5 days from a path results in reducing the time for the total project only 26 days.

The information provided in Table 7 will also prove very helpful to the manager. For example, notice the open column labeled "Actual Date." The manager will first insert the actual project initiation date. Then as each activity is completed and the successor event reached, he will insert the actual date. The actual activity completion time will then be compared with estimates a, m, b, and t_e. If the sum of actual activity times lies between the values of T_E and T_L for the appropriate event, the manager will know that all is on schedule. Table 7 also provides the slacks for each event. The lower the slack value for an event, the more critical the timeliness of the activities preceding it. The variance of each activity time is also provided and indicates, to some degree, the firmness of the relevant activity time estimates. If an activity is becoming more critical and the estimates look relatively uncertain, the manager will probably give it special attention. Finally, in our example, four relatively important check point events (9, 11, 19, and 27) have been established. The probability of reaching each of these events has been calculated. As you might expect, as one proceeds through the network over an increasing number of activities, the probability that a given event will be reached on schedule decreases. A workable solution requires in our example that the completion dates of all paths be equal to or less than 200 days. Consequently, the manager will search the remaining critical path for further time reductions.

A similar problem occurs when a person requests overtime authorization to keep an activity on schedule. Suppose the person responsible for drafting manuscript diagrams (activity M) confronts you with what he considers a tremendous technical problem. His original expected time (t_e) was 7.2 days, but unforeseen problems have stretched that time to 17.2 days, or ten days over the expected time. He asks for overtime help to get back within his original time estimates. As manager, you must decide whether or not to grant this request. According to the project and your PERT network, the answer should be no. You will find why in your network. Activity M only affects total project completion if it delays the activities on the critical path, e.g., Q, EE, and DD. Activity Q cannot begin until after 209.8 days and a delay in activity M of 10 days will enable you to reach event 24 in 128.5 days, long before the 209th day. Thus, you should allocate no overtime.

If the manager feels that the probabilities of reaching an event on schedule are too low, the alternatives are to: a) reduce the T_E's by increasing or reallocating resource application, b) increase T_S by building a greater lag into the committed project completion date, and/or c) reduce activity time variances by insisting on more accurate and certain estimates of a, m, and b.

Management Analysis and Action

Now that we have learned some of PERT's fundamentals, let us review them.

Analysis: If the output shows any negative slack paths and the plan and time estimates seem correct, the contracted project completion date cannot be met unless the plan or some of the time estimates, or both are changed. The first information to suspect is the time estimates, and these demand a rigorous evaluation when they appear on the critical path. At the same time, the planning team should be asked to see if their scheduling plan is correct and preferable to all alternative plans.

With new time estimates and/or a change in the plan, the PERT network is again processed and a second analysis made. The plan is also altered as discussed earlier, i.e., time reductions are sought, resources reallocated, etc. A network is rarely static and will probably change with each analysis, particularly in the earlier stages of managerial planning.

At established intervals, each network should be reanalyzed for a status report. At a prescribed time before an activity, persons engaged in it should receive an "early alert" of the plan, to date. This alert should include mention of any changes in estimated activity times; and if any, what changes in the master network have resulted. If expected times have not changed, a positive statement to this effect should be made.

With this information and that obtained from routine progress reports, the network progresses. Ample information should be available for managerial analysis from daily reports on why some activities were not completed on schedule. However, a complete appraisal would require concentrated action from the planning team, which should view such action as routine.

These procedures should be established for all PERT networks, with those appearing within multiple projects staggered.

Action: What managerial actions should grow from an analysis of network slack paths? First, the manager must determine if the time estimates are valid. If they are, he must remove the negative slack through overtime, resource reallocation, etc. A more important act is to change the order of the activities. Remember, however, that the original network took work to prepare. Hence, changing it is risky.

Zero slack paths are also suspect because they provide no protective time cushion, and a slip in any activity will cause an equal slip in the entire project.

Generally speaking, resources may be transferred to critical paths from positive slack paths. However, having positive slack must not lull one into a false sense of security. It is worth investigating that positive slack decreases with successive network analyses.

If negative slack persists, the planning team should examine the overall network to determine the feasibility of transferring resources from paths with high positive slack. Should this not be

possible, they should recommend to the manager what action to take. They could suggest the need for overtime or other added resources.

Paths with zero and positive slack paths should also be examined. It may be possible to shift resources from the latter to other areas to balance the workload over project duration. Where positive slack persists, the scheduled date for each activity will be between T_L and T_E. However, each path should be examined to allow extra time for "trouble ridden" activities. Most often the manager must judge this matter and the only rule which must be followed is that T_S may not precede T_E nor follow T_L, though it may be the same as either.

When all his action alternatives have been analyzed, the manager may usually shorten network paths in the following ways:

1)   Subcontract some activities.

2)   Skip activities.

3)   Parallel activities.

4)   Add overtime or additional resources.

5)   Change project specifications:

6)   Reevaluate time estimates.

7)   Split activities.

The manager must recognize, however, the risks inherent in each of the above alternatives. For example: a) subcontracting involves a loss of management control over the contracted activity, b) skipping activities (if they are vital) may destroy the project, c) parallel activities complicate management control, d) overtime is costly, e) changing specifications often means reducing quality, f) reevaluating times may necessitate some wild estimating, and g) splitting activities may postpone activity time rather than shorten it.

PERT Time Summary

You will recall from the earlier description that PERT Time:

1)   Is a management tool for defining and coordinating what must be done to accomplish a project's objectives on time. These tasks were effected by our constructing a diagram of the PERT network.

2)   Is a technique that aids the manager but does not decide for him. He uses it to calculate variance, slack, probability, and time estimates.

3)   Is a technique that presents statistical knowledge about the uncertainties faced in completing the many activities associated with a project -- with it we calculated the t_e's, variance, and probability.

4)   Is a method for attracting a manager's attention to latent problems that require decisions and/or solutions. We used it to analyze the PERT network for critical paths and slacks.

5)   Is a method of attracting a manager's attention to procedures for adjusting time, resources, or performance to meet target dates. He does so by analyzing the PERT network for areas of possible resource reallocation.

For project control, it is often important to control more than the time element. PERT Time is well suited to the management control over schedules, but not directly to cost control. For this reason, other systems have been developed along the lines of PERT. Perhaps the most common is PERT Cost. To this new system, we shall now direct ourselves.

PERT COST

Perhaps the single most important factor in developing and accepting PERT Time has been the growth of systems management within this nation's military-industrial complex. Their programs and projects have grown to such size, complexity and breath that an integrative managerial planning and control device like PERT Time is almost mandatory. PERT Time has proven a powerful tool in the kit of managers for planning, coordinating and integrating these large multi-dimensional projects.

When properly implemented, this tool needs no further justification. The PERT network presents a clear picture of project activities and their interrelationships. When times are imposed on the network, the problems of completing program objectives on time become apparent. The manager can scrutinize the activities which critically effect this completion and study the effect of the schedule on workloads. When actual activity times become available, the updated estimates are a dynamic control tool for anticipating adverse results. PERT Time is undoubtedly a tool which, applied with vigor and accuracy, represents a managerial breakthrough in planning and controlling time.

PERT Cost is simply an extension of the PERT Time technique. It assists in planning and controlling program cost expenditures by employing several cost-estimating techniques as monitors to determine variances, i.e., where actual costs are different from planned costs.

The typical agribusiness firm budget is rarely a complete management control system for costs. Although the budget is an important managerial tool, it remains primarily an accounting device which provides historical information for management review and evaluation. Normally this information fails to meet the requirements of daily decision making, i.e., information needed at a time and in suitable form.

Information provided by PERT Cost, on the other hand, is designed precisely for this purpose. It helps the manager find the spots currently creating cost problems or likely to do so in the future. With it, the manager works directly on the more critical problems, prevents cost overruns, anticipates potential problem areas in time to act and identifies those activities from which (or to which) he can divert resources to the more critical program phases. In fact, PERT Cost more effectively integrates normal budget controls with operating decisions.

PERT Cost Background

Most agribusiness managers are faced with a twofold job. Each manager is largely responsible for financially controlling his firm's resources, and he must deliver a product or service at minimum cost on time. The system of program cost analysis synchronized with a PERT Time network must meet these two needs.

In fact, the developers of PERT Cost (Department of Defense and National Aeronautics and Space Administration, DOD and NASA Guide: *PERT Cost,* published by the Office of the Secretary of Defense and the NASA, June, 1962) recognized these responsibilities and designed a three-part system whereby PERT Cost can: a) assist project managers by assigning costs to the working levels of projects in the detail needed for planning schedules and costs, b) evaluate schedule and cost performance, and c) predict and control costs during the project's operational phases. In addition, PERT Cost offered two supplemental procedures: a) the Time-Cost Option Procedure which displays alternative time-cost packages for accomplishing project objectives, and b) the Resource Allocation Procedure which determines the lowest cost allocation of resources among individual project activities to meet the specified project completion dates. For the manager, the basic PERT Cost system provides total financial planning and control by functional divisions, while the two PERT Cost supplements reduce costs.

Finally, PERT Cost estimates indicate a new way to interpret expense budgets. Properly conceived, they can become an integral part of a comprehensive budget program. Yet they differ from conventional expense budgeting in certain respects. For example, from a financial planning and control viewpoint PERT Cost estimates are unconcerned with the normal (discrete) accounting period. In fact, the estimates are activity oriented. Estimates may cut across organizational structures as well as time periods to relate more directly to "things to be accomplished." Furthermore, the focal point of cost accumulation shifts from a department, division, or product to the project work activity. The annual budget may even be bypassed and the end item or objective treated. Finally, from the decision making viewpoint, where the normal cost budget uses volume as the major factor of cost variability, PERT Cost uses activity time. These differences and others shall now be examined in detail.

PERT Cost Framework

To establish a PERT Cost system, one first develops a framework for gathering cost data for each activity. First, all project activities are grouped by function into project subdivisions. The subdivisions are then further broken down into work packages assignable, by area of supervision,

to planning team individuals, i.e., the same planning team described for PERT Time in "Network Construction" on page 7.

The PERT Time network eventually integrates the functionally sub-divided work packages. But for the present, each member on the planning team is asked to prepare cost estimates for the work packages under his supervision. If necessary, further activity work breakdown continues to successively lower levels (called tasks and subtasks) until the size, complexity, and dollar cost of each level is a workable (or an accountable) planning and control unit.

It should be obvious that a cost accounting system so subdivided would comprise numerous accounts. The pragmatic number of account subdivisions will naturally depend upon the detail needed for planning, control accountability, the relative expense of the subdivision, and the relative size of its activity time.

Probably the best rule of thumb is to subdivide network activities until you are assured that each "action" discretely falls within someone's area of responsibility. Next, each subdivision is given an account code so that each network activity cost may be easily accounted.

For example, all activities of our manuscript project might first have been categorized into four functional subdivisions: a) manuscript preparation, b) editing, c) printing, and d) distribution, see Figure 15. Each of these project subdivisions (having now been coded) could now be further divided (and coded) into work packages, tasks, and subtasks to assure their accountability. Figure 15 illustrates a possible activity breakdown structure and coding system which might be used if PERT Cost were applied to our project.

In Figure 15, division continues to the subtask level where the two network activities Y and W first appear, as subtasks under the task of manuscript revisions. All other network activities would appear under other subtasks (not shown). Manuscript revision appears as a task under a writing work package. Other tasks would appear under other work packages. Finally, the writing work package appears as a functional subdivision labeled manuscript preparation.

Each member of the PERT planning team would be responsible for one or more activity for which he would provide time estimates (covered in PERT Time) and cost estimates (to be discussed later). All network activities are then placed in a tree-like activity breakdown structure such as Figure 15. A coding system based on breakdown level and division facilitates a uniform cost accounting of each activity as it occurs; i.e., as each activity cost is expended, it is entered and accumulated under the appropriate subtask, task, work package, and project subdivision. Such an accounting system provides the manager "funds spent vs. funds allocated" information for each level and category of the project as it progresses.

Both cost and time must be planned and controlled from a common framework, so managers may form an accurate picture of project progress and at the same time realistically appraise the consequences of alternative courses of action. The PERT Time network is this common framework to which activity costs are added. The planning team must define the activities carefully so that the network represents cost centers as well as work areas.

Figure 15
PERT Cost Activity Breakdown and Account Code Structure
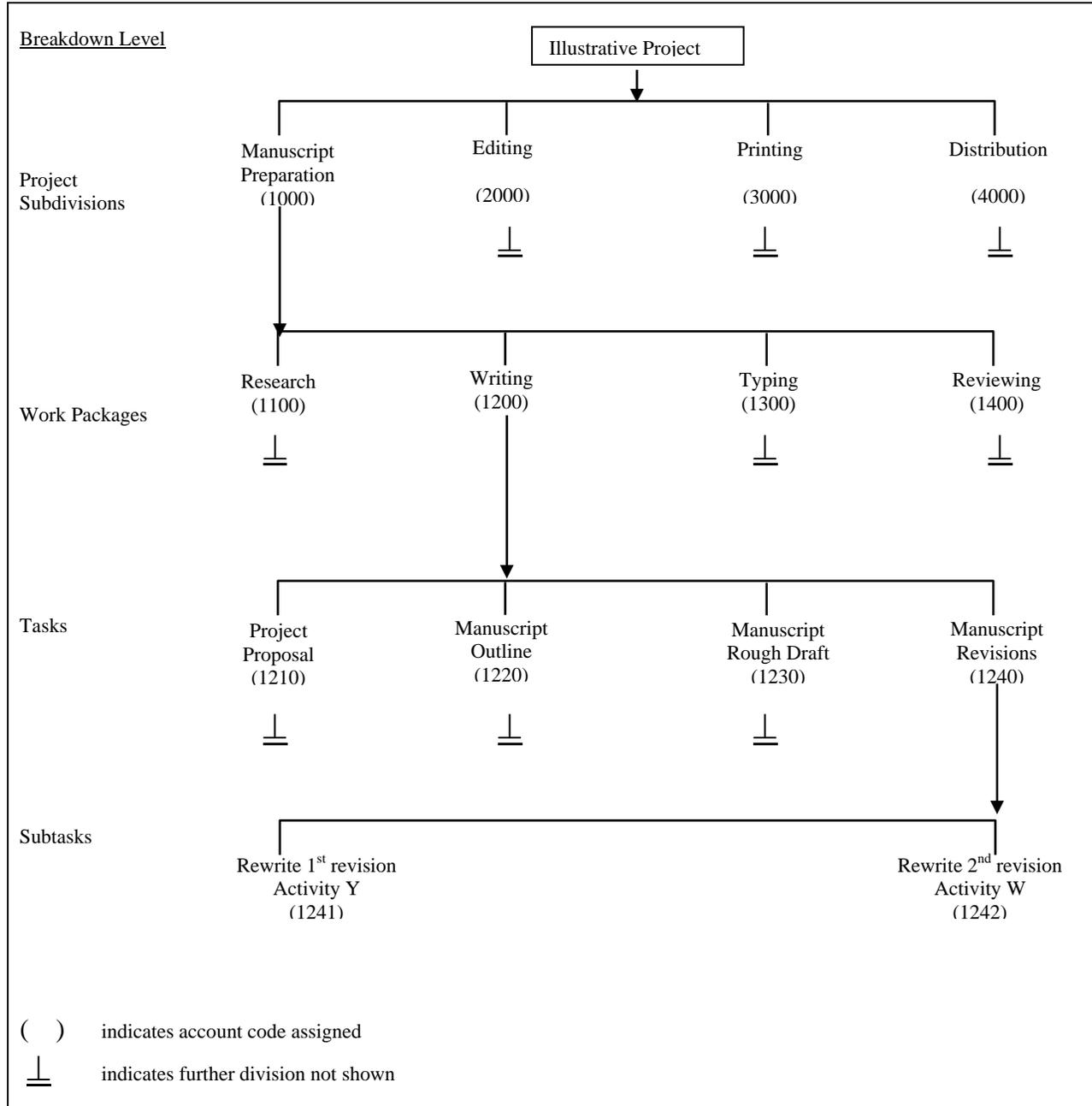
PERT Cost Activity Cost Estimates

Once the network has been constructed, based on the activity breakdown, costs can be estimated. If the breakdown is satisfactory, it will both estimate and accumulate actual cost. Of course, the proper use of PERT Cost, like budgeting, depends on the active participation of the manager and his team.

The activity cost estimates are developed on a sound philosophical and working basis consistent with the manager's information needs. Presently activity cost estimates may take four different forms:

1) a single cost estimate of expected actual cost,

2) three cost estimates combined by formula into an expected cost (similar to determining expected time in PERT Time),

3) optimum time-cost curves (used in conjunction with the Resource Allocation Procedure supplement), and

4) three separate cost estimates (used in conjunction with the Time-Cost Option Procedure supplement).

Each of these four methods of PERT Cost activity cost estimating has as its goal assigning the "best" cost estimates to the network. Yet each method offers management a distinct planning capability.

Single Cost Estimate: A single cost estimate of activity is based upon the sum of cost elements within each activity. These estimates are made by determining manpower, material, and other resources required to complete each activity. The estimates for the direct costs applicable to the network activities are expressed in dollar terms. Indirect cost may be allocated to the individual activity on a pro rata basis or added to the total cost of the project.

Three Cost Estimate: The three cost estimate approach has as its goal determining an "expected activity cost." The advantage of the three cost estimate over the single cost estimate is that the result is subject to probability analysis. As was the case in determining expected time, the three cost estimate formula combines an optimistic, most likely, and pessimistic cost estimate. The expected cost for each activity (Ce) is calculated by the formula:

$$Ce = \frac{Cp + 4C_L + Co}{6}$$

where Cp is the pessimistic estimate, C_L the most likely cost, and Co the optimistic estimate. The standard deviation of the cost distribution can insert probability into the analysis. With this expected cost, the manager cannot, however, assume that he has the optimum time-cost mix for each activity and/or the entire project. Nevertheless, if the cost estimates are realistic, the probabilities of achieving the expected cost can be used for project contract negotiations.

Optimum Time-Cost Curve Estimate: A third approach to activity cost estimates is the optimum time-cost curve concept. Essentially, this concept is differential costing with time (as opposed to volume) as the variable. The intention of this approach is to optimize project time and costs by using optimum estimated activity costs. It assumes the existence of a direct relationship between time and costs for each network activity. A cost curve such as that shown in Figure 16 is developed for each activity.

Figure 16
Cost Curve — Activity W
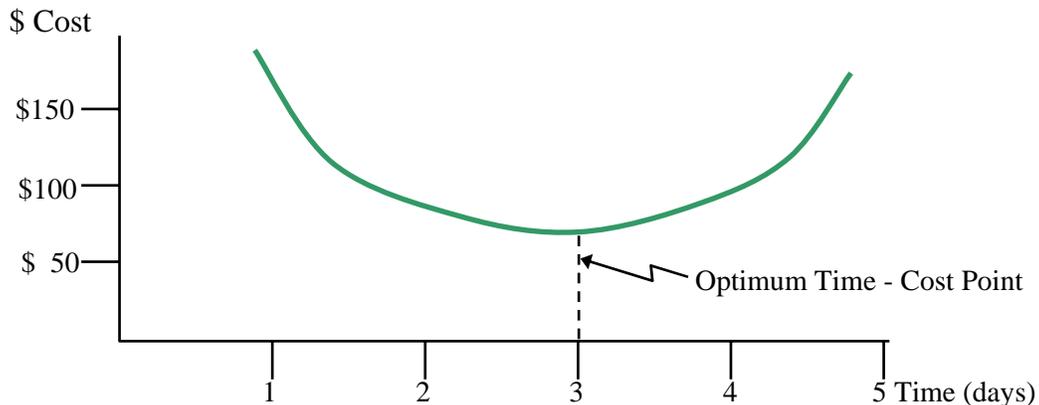
Network schedules may then be modified to obtain the lowest project cost commensurate with an established project completion date. Other related questions can be expected such as: How long will project completion take with a fixed time period? In theory, this method of estimating activity costs is undoubtedly superior to either of the earlier noted methods. It has one major drawback, i.e., the lack of past project and activity cost data makes it impossible to construct the necessary cost curves.

Resource Allocation Supplement: Because developing continuous time-cost curves for all activities is so difficult, the Resource Allocation supplement to PERT Cost was evolved. This supplement is composed of a variation of continuous time-cost curves which can be used to plan and control a group of important activities representing only a minor portion of a project network. This supplement method is also based on the concept that activities are subject to time-cost trade-offs. The Resource Allocation supplement method of cost estimation is described below in a simple illustration and a series of procedures:

Step 1: Construct network of activities in which you are interested.
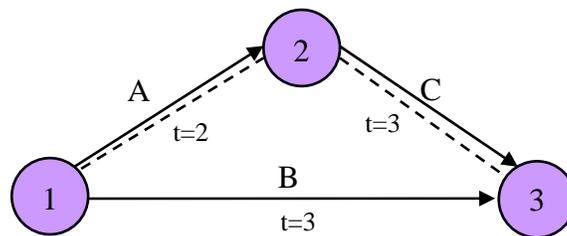
(Assignment: Reach event
3 in 4 days)
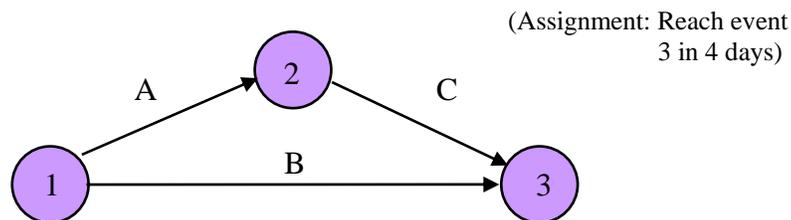
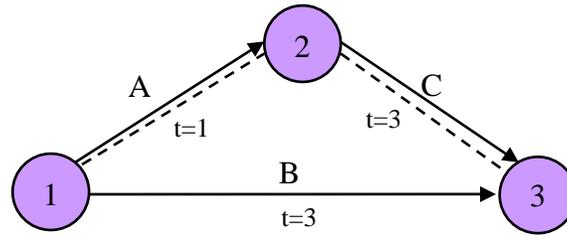Step 2: Obtain alternative time-cost estimates for each activity.
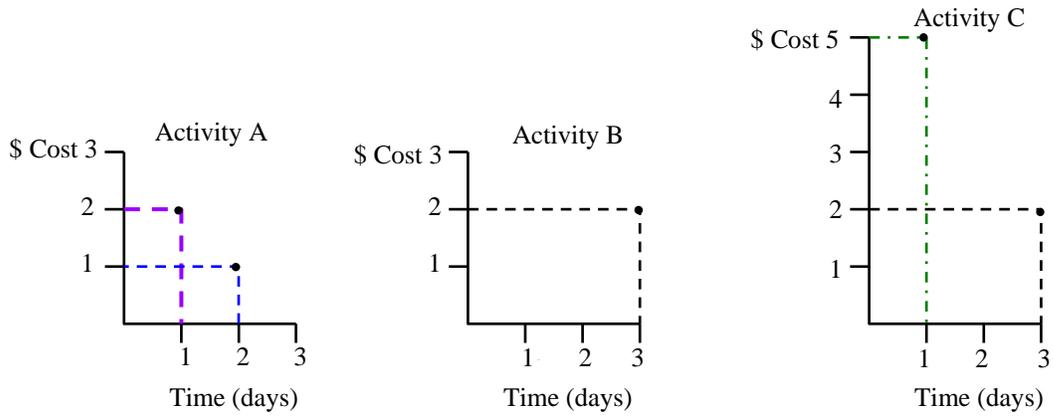
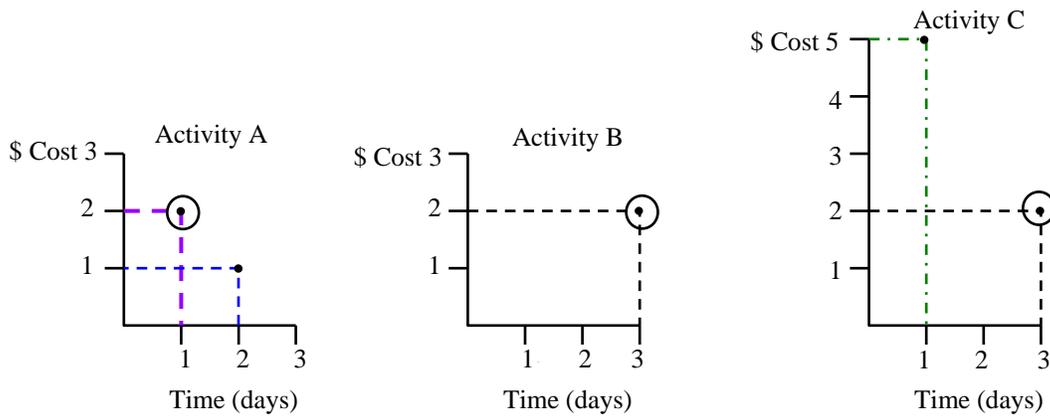Step 3: Select the lowest cost activity for each activity.
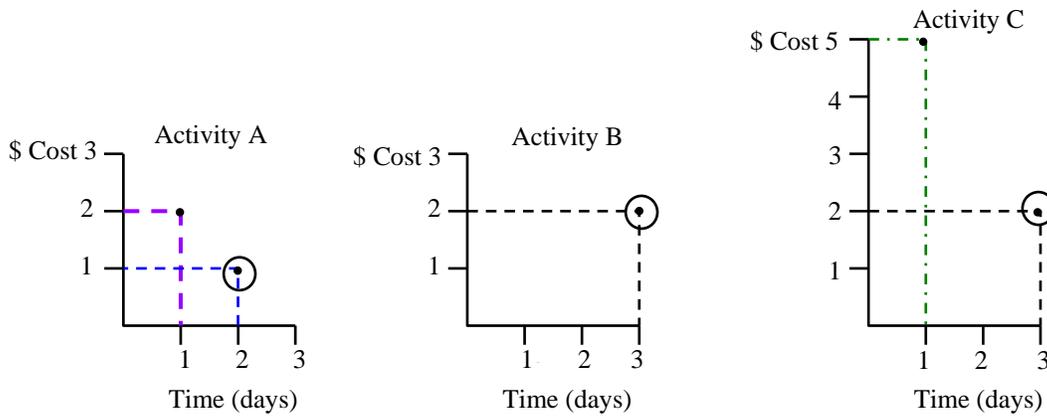
Step 4: Calculate critical path and compare with directed completion date.

Step 5: Adjust critical path to meet directed completion date at lowest possible costs.


(═══════ critical path takes 4 days, directed completion date is 4 days, scheduling change resulted in an increase in cost from $5 to $6.

Summary: In the Resource Allocation supplement procedure, one can determine how to accomplish a project by a specified date at minimum cost. The critical path is from event 1 to event 2 and from event 2 to event 3 since it requires 5 days at absolute minimum costs. However, our assigned completion date is 4 days from the beginning. From our activity time-cost charts, one finds that cutting activity A one day doubles its cost. However, since shortening the activity time for activity C would result in an even greater cost increase, we choose to shorten activity A's time.

An alternative to overcoming the shortcomings of continuous cost curves is a linear cost function based on two time-cost relationships. The cost and time expenditures for each activity are estimated for only two conditions: normal and crash. The normal point is the minimum activity cost and corresponding time; whereas the crash point is the minimum time in which the activity can be performed and the related cost. As shown in Figure 17, a linear function is assumed to exist between these two points.

Figure 17
Linear Time-Cost Relationship

Cost

Crash Point

Normal Point
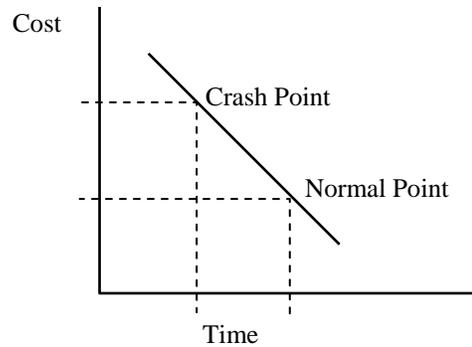
Time

As you may already suspect, you will face the problems of realistic estimates, discretionary costs, stair-stepped cost functions, incorrect correlation between time and cost, and external factors using this supplement method. Nevertheless, the simplicity of the procedure is probably its greatest virtue. For example let's consider the following PERT network, see Figure 18.

Figure 18
Illustrative PERT Network

E=7

4

A $t_e=2$

G $t_e=4$

D $t_e=4$

E=0   1

B
$t_e=3$

2   E=3

F $t_e=7$

5   E=11

C
$t_e=6$

E $t_e=2$

H $t_e=3$

3

E=6
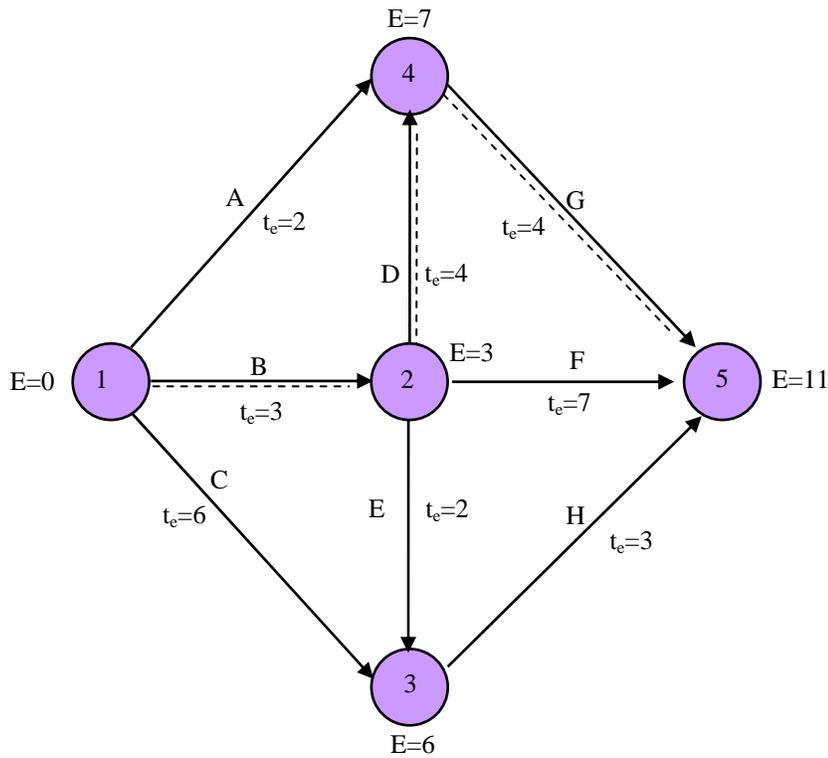
where: t_e is the estimated time per activity,
E is the expected completion time of an event,
and ----- is the critical path.

The critical path of this network is eleven days. To accelerate the project completion one day, activities B, G, or D must be condensed one day. Based upon cost curves computed on a normal-crash basis, the table of costs appears below, see Table 8.

Table 8
Activity Costs by Normal -- Crash Procedure

	Normal		Crash		Acceleration Cost Per Day*
Activity	Days	Cost ($)	Days	Cost $	
A	2	80	1	130	50
B	3	70	1	190	60
C	6	110	5	135	25
D	4	60	3	100	40
E	2	90	1	100	10
F	7	85	6	115	30
G	4	105	3	175	70
H	3	50	2	70	20
Totals		$650		$1,015	

* This is also referred to as the cost slope (m) and calculated as:

$$m = \frac{\text{crash cost - normal cost}}{\text{normal time - crash time}}$$

Since activity D costs $40 to accelerate one day, whereas activities G and B cost $70 and $60 respectively, accelerating activity D is the least expensive way to meet a scheduled completion date of ten days. The total cost of completing the project in ten days is $690, i.e., $650 + $40. Notice, however, that by reducing activity D to three days, activity F has now entered the critical path. What would it cost to reduce the scheduled completion time to nine days? In essence, we now have two critical paths: a) B, D, G, and b) B, F. For one-day reductions, B costs $60, G costs $70, F costs $30, and D can be reduced no further (or it has prohibitively high reduction costs). To meet a nine-day completion date, both G and F would have to be reduced by one day (at a total cost of $100) or B be reduced by one day (at a total cost of $60). Therefore, for the reduction to nine days, the total cost would be $750, i.e., $650 + $40 + $60. The costs of further reductions in completion times could be similarly determined until the manager had information on the marginal cost (that is the increase in total cost) of reducing the project completion time from a normal basis to a crash basis.

Time Cost Option Procedure Supplement: The single estimate of expected activity cost and the three cost estimate formula methods do not suggest a more efficient alternative plan. The continuous cost curve concept does, but requires sophistication in cost analysis or supposition. The Time-Cost Option Procedure supplement, on the other hand, recognizes that a single

estimate will normally be used for initial project contract procedures, but that additional data are needed to determine the time that might be saved by spending more money, or the money that could be saved by extending the contract time. The procedure requires a gradual progression through three plans as shown in Figure 19.

Figure 19
Time-Cost Option
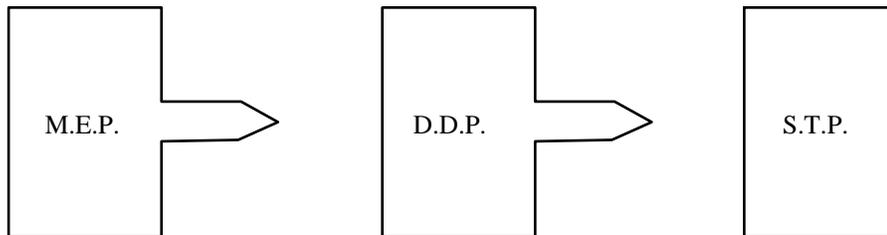
Where:

M.E.P. = Most Efficient Plan: This is the network plan that will meet the technical requirements of the project making the most efficient use of existing resources, i.e., it is the plan that would be chosen without any budget or time constraints.

D.D.P. = Directed Date Plan: This is the network plan developed to meet the technical requirements of the project by a specified completion date.

S.T.P. = Shortest Time Plan: This is the network plan that will meet the technical requirements of the project in the shortest (under crash conditions) time.

Since the desired plan is the M.E.P., any project study should begin here. The M.E.P. must then be modified to achieve the projects objectives by a specified completion date. The M.E.P., when altered by contracted completion dates becomes the D.D.P. The D.D.P. is then revised to obtain the S.T.P.; i.e., the activities that have not changed in evolving the subsequent plans will utilize cost estimates from the M.E.P., and new cost estimates are derived for those activities expected to increase or decrease because of the modifications.

The methods of estimating cost discussed above are current. Combined with a sound approach to breaking the activity down and coding the structure, they aid managerial planning. PERT Cost is as yet only a planning tool, i.e., it does not link planning and control. For control to occur, the manager must compare actual vs. estimated activity cost expenditures. It is often the accountant, taking a running account of activities, who provides the manager with this control information. Such information normally appears in the form of a series of PERT Cost reports.

PERT Cost Reports

Reports evolving from a PERT Cost system must be interrelated and coordinated to unify the procedure for presenting decision making information. Furthermore, the procedure should present information relevant to all management levels. Since the various PERT Cost reports evolve from the same basic data, no opportunity exists for individual biases. The manager who uses the data may rest assured that all data used for reporting is being (or has been) used for PERT planning and control. Furthermore, the cost reports not only identify the project's problem areas, but through the account coding system, relate each problem to a particular supervisor and level on the activity breakdown chart.

While it is important for each agribusiness firm to design its own PERT Cost reports according to its informational needs, we might profit by briefly discussing several more-or-less standard PERT Cost reports. Each document, in one way or another, relates to the PERT Cost system objectives and, through minor modification, might prove adaptable to several different types of agribusiness firms.

The five basic PERT Cost reports of importance are: a) the Management Summary Report, b) the Manpower Report, c) the Milestone Report, d) the Predictive Report, and e) the Cost-of-Work Report.

The reports listed above do not represent the total information output of PERT Cost. Nor are they the only classifications of PERT Cost. Rather, they are one way to provide the timely information required for intelligent decision making. Other reporting systems are probably equally well suited. They need only be integrated and interrelated with PERT Cost's characteristics and management's information requirements.

The Management Summary Report: The objective of the Management Summary Report (see Table 9) is to present an over-all time and cost schedule for both the project as a whole and each of its activities. It highlights the problem areas requiring the manager's special attention and guidance. It singles out time schedule slippage and cost overruns. It indicates the amount of slippage or cost variation that might delay project completion. It indicates the actual cost overrun or underrun to date by comparing the estimated with the actual work cost. In so doing it anticipates the cost variance by analyzing the original project cost estimate in relation to its actual plus estimated costs.

When the manager receives this kind of a summary report on an ongoing project, he is better able to make correct decisions at the correct time, thereby controlling the project at its most profitable level. Since this report can be prepared for all levels of project supervision, we find that we have a comprehensive reporting system for integrating management information for decision making.

Manpower-Loading Reports and Displays: The PERT Cost system can be operated without the Manpower-Loading Report on activities being accomplished in the network. However, without this report the manager lacks the necessary information for making a decision on manpower allocation. Such information might help him shift skilled resources from an activity with positive slack to one with negative slack.

Table 9

MANAGEMENT SUMMARY REPORT	Project:		Program:		Division:		Period Covered By Report:
	Contractor:		Contract #:			Date of Report:	

| Activity by account code no.	Cost of Work						Duration of Work			Remarks
	Work Perf. to date ($)			Totals at Completion ($)			Scheduled Activity Completion Date	Actual Activity Completion Date	Slippage (Slack)	S_L=slippage S_A=slack O =cost overrun U =cost underrun
	Original Est.	Actual Costs	Overrun (Under-run)	Contract Est.	Revised Est.	Overrun (Under-run)				

The Manpower-Loading Report (see Table 10) shows the plan for manpower assignments within the project and is usually prepared for each administrator or supervisor. The report is used to plan personnel allocation and establish the need for additional employees, overtime, rescheduling, or whatever necessary to make the project operate efficiently.

Table 10

MANPOWER-LOADING REPORT	Project:		Program:		Report Date:
	Contractor:		Contract #:		
Date Manpower Required	Performing Unit	Activity Acct. #	Estimated Man-Hours Allocated	Skill Required	Activity Slack (weeks)

The Manpower-Loading Display (see Table 11) shows the total manpower requirements, while the Manpower-Loading Report shows the allocation of man-hours to the various activities (or groups of activities) as shown by their accounting code numbers. The report is the means by which the manager can balance out manpower, i.e., shift it to situations critical to the project, thereby, utilizing the available skills to advantage. The display makes it possible to minimize overtime by allocating manpower from one month to another or by rescheduling slack activities. This will permit activities to be accomplished with available manpower and, since slack activities are not on the critical path, do so without delaying project completion.

Table 11

MANPOWER-LOADING DISPLAY	Report Date:	Performing Unit:	Skill
	Project:	Program:	
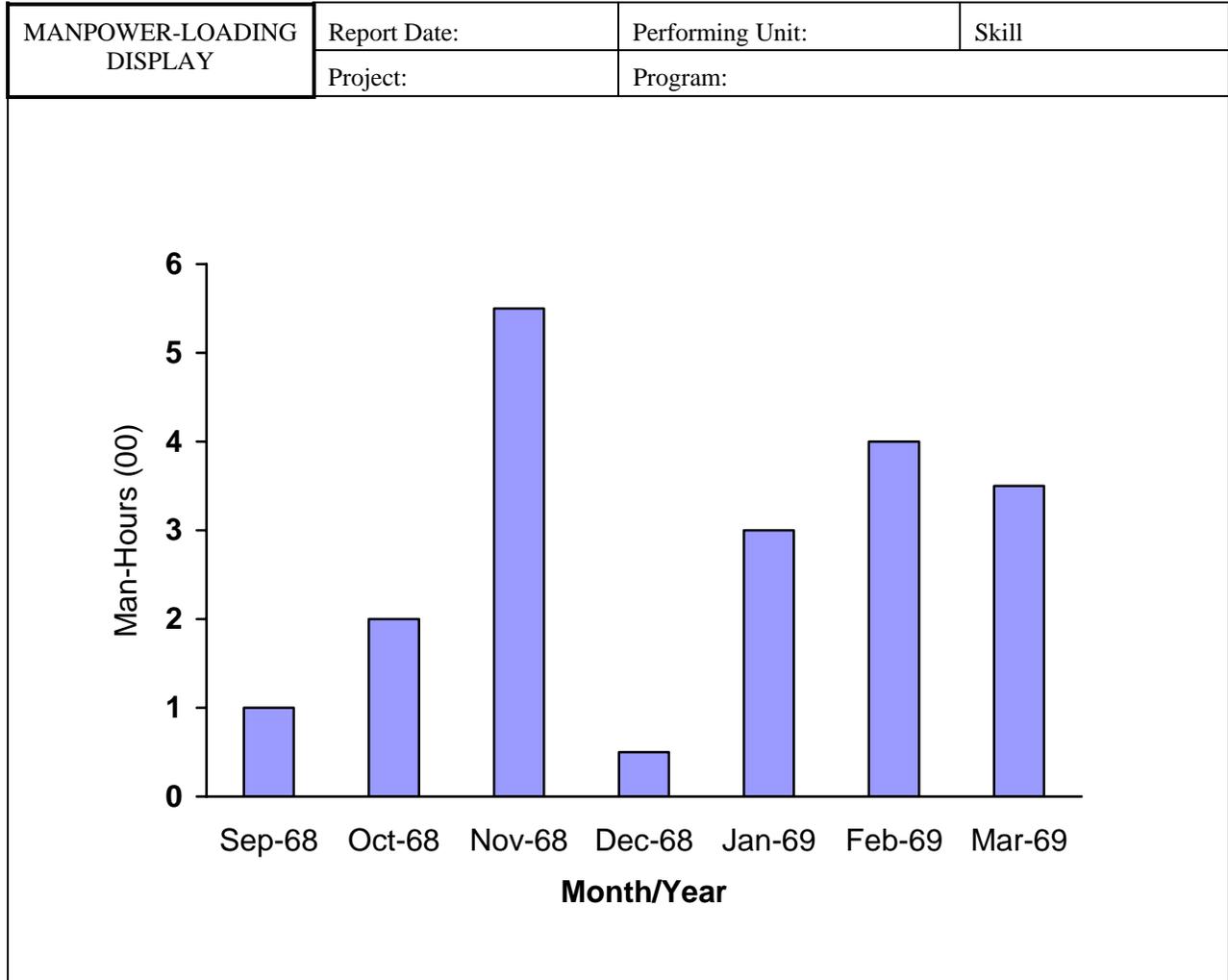
Cost-of-Work-Reports: The Cost-of-Work-Report (see Figure 20) develops the information necessary for the following:

    a) Budgeted costs --   the money required to accomplish the project.

    b) Committed costs --   the actual costs committed or expended to date.

    c) Cost performance --   the estimate for the work performed to date based on original estimates.

    d) Projected costs to complete --   the estimated cost to complete the project based on actual costs to date and estimated costs for the balance of the project.

Figure 20

COST OF WORK REPORT

| Project: | Program: |
| Report Date: | |

Time Now

80
75
70
65
60
55
50
45
$(00)
40
35
30
25
20
15
10
5
0

2/68 3/68 4/68 5/68 6/68 7/68 8/68 9/68 10/68 11/68 12/68 1/69 2/69 3/69 4/69

Month/Year

Projected Overrun $500

Schedule Slippage 6 weeks

Budgeted $

Under Budget $1200

Actual $

Overrun $700

Contract Estimate for Work Performed

Budgeted $ = planned rate of expenditure

Actual $ = expenditures and commitments to date

The Cost-of-Work-Report analyzes the original project estimate to see whether the actual costs to date are in line with the estimates. As shown, the Cost-of-Work-Report normally appears as a graph and presents its comparison in a simple summary form.

Predictive Reports: Predictive reports are normally labeled cost or schedule outlook reports (see Figures 21 and 22). They graphically predict a cost overrun or a time slippage on the over-all project. The information is accumulated and plotted to show trends. Monthly projections of the extra time and cost necessary to complete the project enable the manager to judge his effectiveness in planning and control. The predictive reports, therefore, help a manager to project with precision into the future as well as to review his past.

Milestone Reports: A milestone is a selected event in the PERT network that represents a major accomplishment toward project completion. It is both a target and benchmark of progress. It helps managers determine how well they are doing. In some instances it may be desirable to include this type of information in the PERT Cost Management Summary Report, but on other occasions separate milestone reports can be prepared. These milestone reports are generally submitted along with a written analysis explaining actions accomplished on schedule and actions planned. Because they give the manager a feeling for the whole project, they generally cast light on how all activities fit into the total project.

The Value and Uses of PERT Cost

Perhaps PERT Cost's greatest value is its ability to directly relate a) project action vs. schedule performance to b) project action vs. cost performance. Only through this time-cost relationship can a manager find answers to such questions as:

a) How much has the project overrun its cost?
b) How far behind schedule is it?
c) What activities are creating cost and schedule problems?
d) What time-cost trade-offs exist within the project?

PERT Cost can provide a manager with information he would not otherwise obtain. His problem is to overview simultaneously a large number of activities and PERT Cost Reports. However, PERT Cost can provide this overview to whatever depth and degree his time schedule, skill, and ability can accommodate. Most important of all, its combined schedule and cost mechanisms provide the type of information most relevant to the project and most important for intelligent action.

PERT Cost does not provide project planning and control; only the manager can actually provide these. However, it does indicate how, where, and when planning and control is needed in time for the manager to take remedial action.

In addition, PERT Cost is useful throughout the project. Also, the manager can rest assured that the project planning team, the production staff, and all levels of management have compatible information on the project's cost and schedule performance.
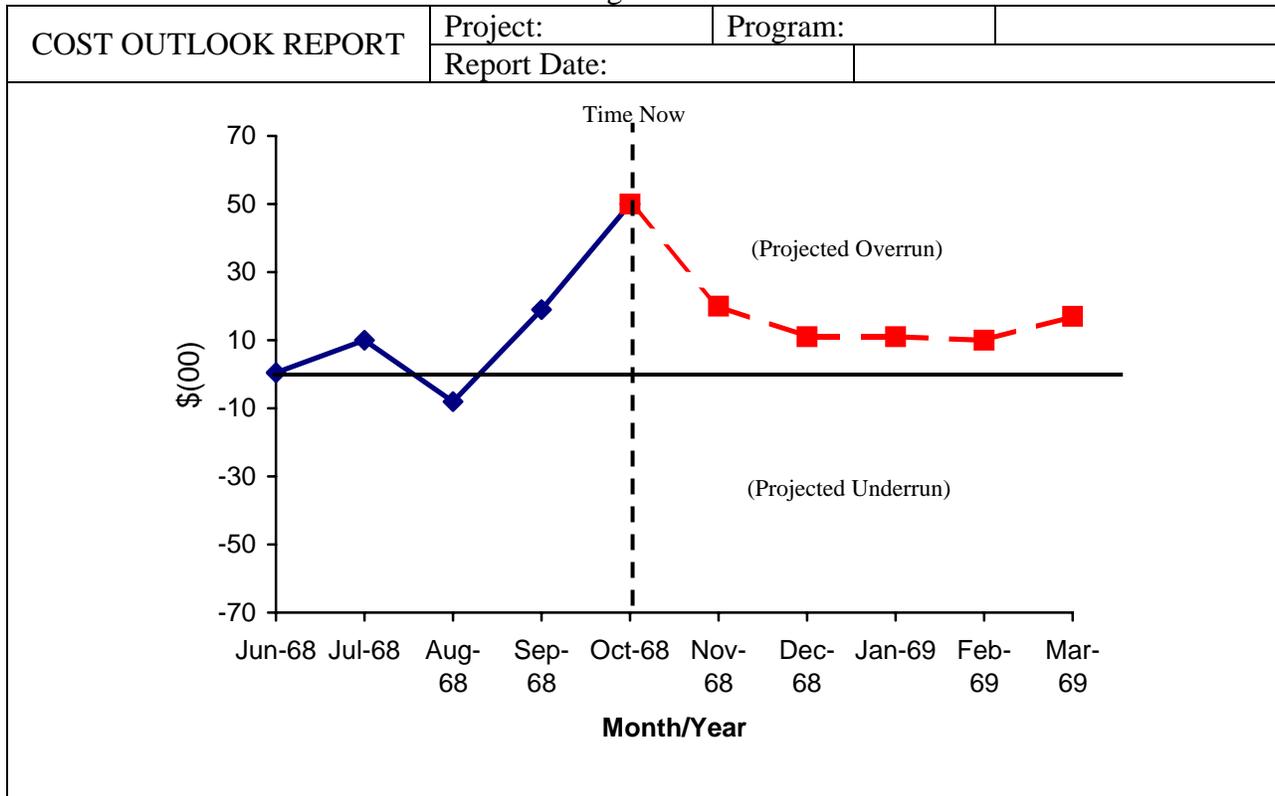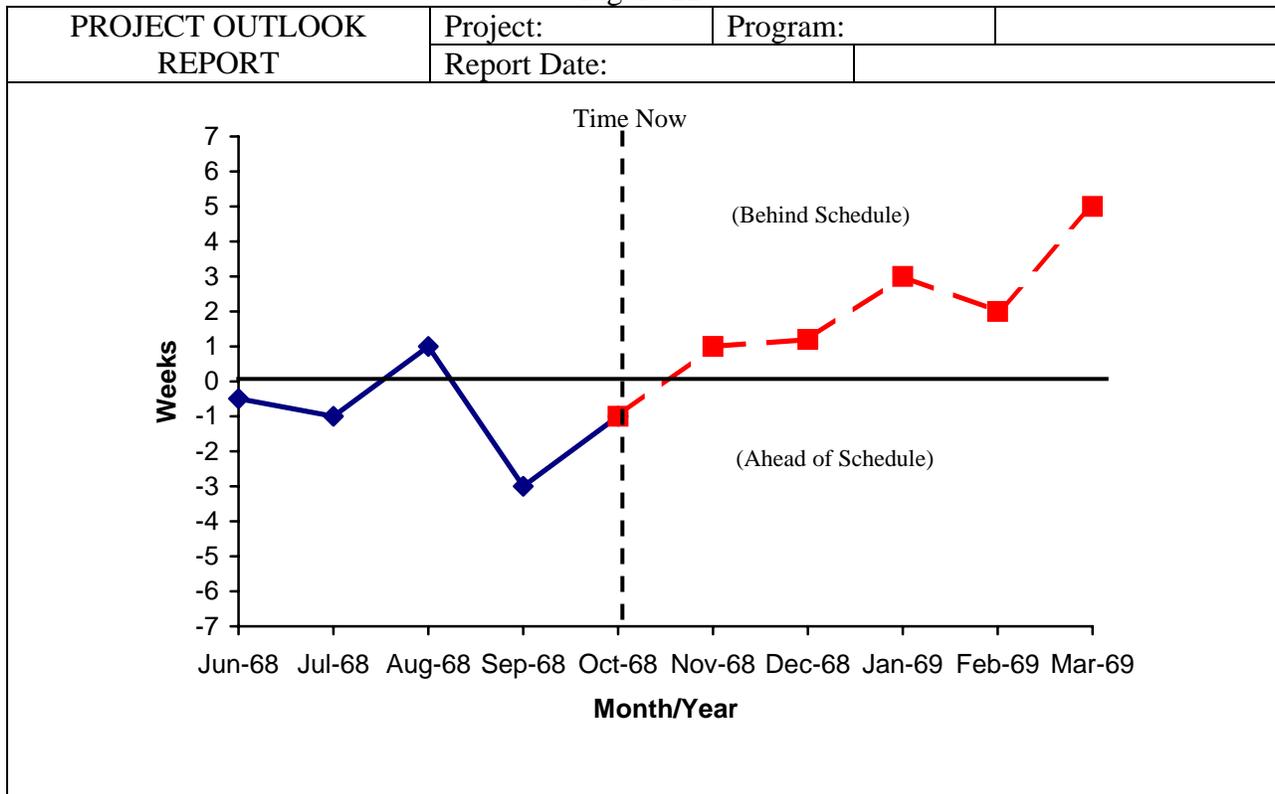
Figure 21

COST OUTLOOK REPORT	Project:	Program:	
	Report Date:		

Figure 22

PROJECT OUTLOOK REPORT	Project:	Program:	
	Report Date:		

The use of PERT Cost unquestionably has many other advantages. It highlights the time-cost interrelationships and the financial effects on the project of potential changes in resource application and/or time schedule. PERT Cost also quantifies the activities to be performed and assesses the adequacy of present funding for meeting total project objectives. It compares the time schedules and resource needs of different functional divisions or of different contractors.

By integrating PERT Time and PERT Cost, one can determine whether the various members of the planning team are meeting their schedule commitments, cost estimates, and technological performance standards. If inadequacies are noticed, the manager may consider how resources could be recombined to minimize costs. Finally, in measuring the progress of a project, the sum of actual costs to date may be compared with the funds authorized and the estimated cost of project completion. Such a comparison will indicate potential cost overruns or underruns and reveal those activities requiring special cost control action by the manager.

PERT Cost Limitations

The discussion so far might lead one to believe that PERT Cost creates only benefits and no problems. Unfortunately this is not the case. Despite all its attributes, it has some basic problems which managers should recognize:

a) It may be difficult to adapt PERT Cost activity costs to fiscal accounting practices.

b) It may become difficult to allocate project overhead costs to the activities when several functional divisions are concerned with simultaneous activities.

c) Historical information for making activity cost assignments may be lacking for non-repetitive projects.

d) Firm personnel may lack the sophistication in cost analysis needed to optimize costs.

e) Since time and cost are directly related, and since inexperienced planners tend to be pessimistic about meeting time schedules, cost estimates are often artificially padded to allow for a margin of safety.

f) Because PERT deals with so many uncertainties it is difficult to extrapolate knowledge from previous experience. Thus, the assumption that a particular decision will result in a least-cost performance may be incorrect.

g) It is often too easy to subject cost estimates to precise mathematical measurements; the results giving the user a false sense of security. This does not imply that costs are not predictable for discrete activities or that PERT Cost is not a sound framework for cost control. Nevertheless, the difficulty of obtaining reliable cost estimates certainly limits the system's overall effectiveness.

PERT Cost is not, after all, a complete departure from earlier planning and control techniques. Its elements existed before under other names and systems. Most PERT Cost components, therefore, are evolutionary, i.e., concepts of managerial planning and control have simply been refined and linked within a more compatible and effective system.

Summary

Despite the limits listed above, PERT Cost is a new and useful tool for agribusiness managers. It is designed to bring the managers closer to a total system's approach to planning and control. Just as PERT Time provides timely information which helps achieve project objectives more rapidly, so PERT Cost provides financial information which facilitates achieving these objectives promptly, efficiently, and economically. Just as the manager is forced by PERT Time to examine more closely schedule dates and activity performance, so PERT Cost forces the manager to be more cognizant of the resources used to meet schedule dates and maintain activity performance.

Like other planning and control techniques, PERT Cost is no manager's panacea. Its ultimate value and usefulness depends directly on the relevancy and accuracy of the data fed into the system, which, in turn, depends on the skills, abilities, and desires of the manager and other supervisory personnel.

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Glossary of PERT Terminology

Account Code Structure -- the numbering system used to assign summary numbers to elements of the activity breakdown structure and charge numbers to individual divisions.

Activity -- an element of a project which is represented on a network by an arrow. An activity cannot be started until the event preceding it has occurred and may represent a process, task, procurement cycle, job, or a waiting time.

Activity Time -- estimates of elapsed time necessary to complete an activity in a specified manner. Three time estimates may be given for each activity (optimistic, most likely, and pessimistic), or only one (most likely) may be required.

Actual Date (T_A) -- the calendar date on which an event occurred or an activity was completed.

Anticipated Slippage -- a future anticipated delay or non-accomplishment of an event in a network.

Beginning Event (predecessor event) -- an event which signifies the beginning of one or more activities on a network.

Burst Point -- that point in a PERT network where two or more activities may begin as a result of having completed a common predecessor event.

Constraint -- the relationship of an event to a succeeding activity wherein the activity may not start until the event preceding it has occurred. The term is also used to indicate the relationship of an activity to a succeeding event wherein the event cannot occur until all activities and events preceding it have been completed.

Critical Path -- that particular sequence of events and activities in a path that has the greatest negative or least positive slack; therefore, the longest path through the network.

Dangling Event -- any event other than the start or end events that has either no predecessor or no successor.

Directed Date for an Event (T_D) -- Date for a specific accomplishment formally directed by the contracting authority. A schedule date (T_S) which has been formally specified by contracting authority as the date of project completion.

Dummy Activity -- a zero-time activity used to indicate a constraint and no work is performed.

Earliest Completion Date (S_E) -- the earliest calendar date on which a work effort (activity, work package, or summary item) can be completed. This date is calculated by summing the scheduled elapsed time (t_S) for activities on the longest path from the beginning of the program or project to the end of the work effort, and then adding this sum to the calendar start date of the program or project. For distant time effort where scheduled elapsed times (t_S) have not been established, expected elapsed time (t_e) will be used to calculate S_E.

Earliest Expected Date (T_E) -- a summation of the expected times (t_e) from the initial event of the activities preceding the event, transposed to a calendar date. Where more than one activity precedes an event, the date used is the latest one.

Earned Hours -- the number of equivalent hours of work accomplished in relation to the standard.

End Event -- that event which signifies the completion of a path through a network.

Estimate-to-Complete -- the estimated man-hours, cost and time required to complete a project (includes applicable overhead except where only direct costs are specified).

Event -- a specified, definable accomplishment in a program plan, recognizable at a particular instant in time. Events do not consume time or resources.

Expected Elapsed Time (t_e) -- the estimate of the time an activity is expected to consume. The t_e value can be determined from the formula, $t_e = \dfrac{a + 4m + b}{6}$, or can be determined by analysis of the specific task and the resources available for it.

Integrated Network -- a union of two or more detail networks. The number of events activities in the integrated network is equal to the total shown on the detail networks being integrated.

Interface Event -- an event which signals the necessary transfer of responsibility, end items, or information from one part of the plan to another. Examples of interface events are the receipt of an item, or the release of engineering drawings to manufacturing, or release from contractor.

Joint Cost Activity -- one of several cost estimates in an activity breakdown whose total cost does not warrant further subdivision for cost control purposes. Joint Cost Activities include also those activities that share resources in such a way that is impractical to allocate them to individual activities.

Latest Allowable Date (T_L) -- the latest calendar date in which an event can occur without delaying the completion of the project. The T_L value for a given event is calculated by subtracting the sum of the expected elapsed times (t_e) for the activities on the longest path between the given event and the end event of the program from the latest date allowable for completing the program. T_L for the end event in a program is equal to the directed date (T_D) of the project. If a directed date is not specified, $T_L = T_E$ for the end event.

Latest Completion Date (S_L) -- the latest calendar date on which a work effort can be scheduled for completion without delaying the completion of the project. This date is calculated by summing the scheduled elapsed times (t_S) for activities on the longest path from the end of the work effort to the end of the project, and then subtracting this sum from the calendar end date of the project. For distant time effort where scheduled elapsed times (t_S) have not been established, expected elapsed times (t_S) will be used to calculate S_L.

Latest Revised Estimate -- the sum of the actual incurred costs plus the latest estimate to complete as currently reviewed and/or revised (including applicable overhead except where direct costs are specified).

Loop -- a network error condition which occurs when an event succeeds itself.

Man-Hours -- the common unit of direct labor used in reporting actual labor expenditures. (In PERT/COST man-months, man-weeks, number of men, etc., are converted to man-hours for the rate extension of estimates.)

Milestone -- key events selected by project management whose status serves as an indicator of program progress. Milestones are synonymous with events in a network.

Most Likely Time Estimate(s) -- the most realistic estimate of the time an activity might consume. This time would be expected to occur most often if the activity could be repeated numerous times under similar circumstances.

Network -- a flow diagram consisting of the activities and events which must be accomplished to reach the project objectives shown, their planned sequences of accomplishment, interdependencies, and interrelationships.

Node -- an event with two or more preceding events.

Optimistic Time Estimate (a) -- the time in which the activity can be completed if everything goes exceptionally well. It has no more than one chance in 100 of being realized.

PERT/COST -- a modification of the basic PERT technique which provides for the collection of actual cost data at any level of indenture in an Activity Breakdown Structure down to and including the functional breakout of the activities.

Pessimistic Time Estimate (b) -- an estimate of the longest time an activity would require under the most adverse conditions barring acts of God, strikes, floods, fires, etc.

Planned Cost -- the approved planned cost for an activity. This cost, when totaled with the planned costs for all other activities, results in the total cost estimate, committed under contract, for the project.

Projected Overrun or Underrun -- the planned cost minus the latest revised estimate for an activity. When planned cost exceeds latest revised estimate, a projected underrun condition exists. When latest revised estimate exceeds planned cost, a projected overrun condition exists.

Resource Code -- a code to indicate the type of resource applied to the activity by the performing organization.

Scheduled Completion Date (T_S) -- a date assigned for completion of an activity for purposes of planning and control within an organization. Where no specific date is assigned, $S_E = T_S$.

Scheduled Elapsed Time (t_s) – the period of time assigned for performing an activity.

Slack -- the difference between the latest allowable date and the expected date (T_L - T_E), or the difference between the latest completion date and earliest completion date (S_L - S_e). Slack may be positive, zero, or negative.

Standard Deviation of an Activity -- a measure of uncertainty of the expected elapsed time for an activity, calculated when using three time estimates. It is computed from the formula, $\dfrac{b-a}{6}$.

Summary Item -- an item appearing in the activity breakdown structure.

Summary Level -- any level in the activity breakdown structure.

Summary Account Code Number -- a number which identifies an item in the activity breakdown structure.

Transaction Code -- a one-digit numeric character which describes the nature of the action to be taken to process data for the input card (add, delete, etc.).

Value (Work Performed to Date) -- the planned cost for completed work, including that part of the work in process which has been finished. This value is determined by summing the planned cost for each completed activity. If an activity is in process, the part of its total planned cost which applies to work completed is approximated by applying the ratio of actual cost to latest revised estimate for that activity.

Variance of an Activity -- the square root of the activity standard deviation, $\left(\dfrac{b-a}{6}\right)^2$.

Activity Breakdown Structure -- a family tree subdivision of the project, beginning with the project subdivisions and then subdividing these objectives into successively smaller end item subdivisions. The activity breakdown structure establishes the framework for a) defining the work to be accomplished, b) constructing a network plan, and c) summarizing the cost and schedule status of a project for progressively higher levels of management.

Work Package -- the work required to complete a specific job or process, such as a report, a design, a documentation requirement or portion thereof, a piece of hardware, or a service. A work package may consist of one or more cost significant activities. The content of a work package may be limited to the work which can be performed by a single operating unit in an organization or may require the contributing services of several operating units. The overall responsibility for the work content of a work package should be assigned to a single organization or responsible individual. It is the lowest level of identification of costs to work performed and is represented by a charge number related to a single summary number. The work package couples to the cost accounting system through the account code number, and to the PERT network through the beginning and ending event numbers of activities in the package.

Case Problem

Master-Mix Processing, Inc.

Assume that you are the manager of Master-Mix Processing, Inc. This corporation is involved in the processing, packing, packaging, and distribution of a fresh-frozen consumer pack of mixed vegetables. The firm has operated fifteen years under your supervision and is financially sound.

Because of recent technological advancements in the food processing industry, a new quick-freeze packing line has been developed and is now being used by two of your major competitors. The new quick-freeze process allows for increased packing efficiency and greatly improves the quality of the product. In order to maintain your share of the fresh-frozen vegetable market, your board of directors decides that the newly-developed quick-frozen packing line should be purchased and installed before the up-coming harvest season. The old packing line is quickly dismantled and removed from the plant during the off-season.

Only 23 weeks are available until the vegetable harvest for this year begins. You call all your supervisors together and form a PERT planning team. You inform this team that their project objective is to complete the purchase, receipt and installation of the new packing line at the lowest possible cost in time to accept this year's crop.

The planning team begins work immediately and decides that the project assigned to them will involve the following PERT activities and events:
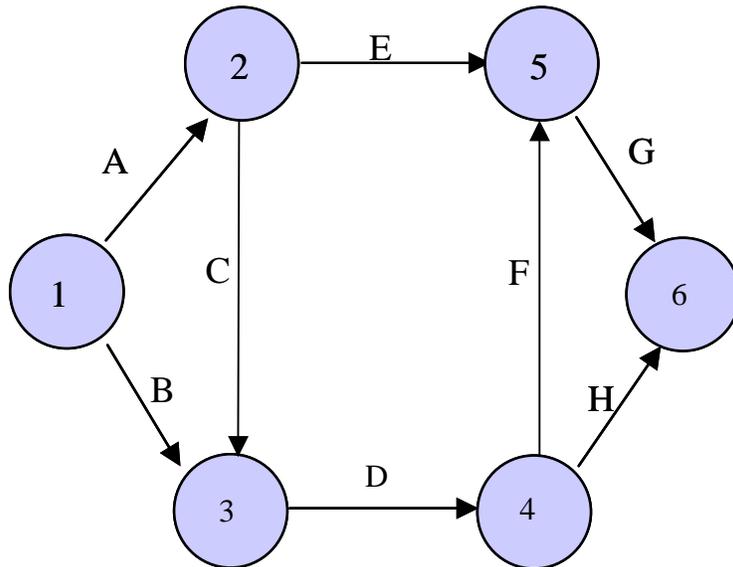
Activities:

A. Obtain needed financing from local bank.
B. Submit equipment order and specifications to manufacturer.
C. Submit payment (purchase contract) to manufacturer.
D. Delivery of packing equipment.
E. Negotiate a contract for the installation of new equipment.
F. Replacement of equipment parts not meeting specifications or damaged in transit.
G. Installation of new equipment.
H. Federal health and safety inspection of new equipment and its installation.

Events:

1. Installation project begun.
2. Financing arrangements completed.
3. Submission of order and payment completed.
4. Delivery of new equipment completed.
5. Acceptance of equipment and installation contract completed.
6. Completion of installation and inspection.

After still further discussion, the planning team agrees that each activity and event should appear as shown in the construction of the following PERT network, see Figure I

Figure I
Master-Mix PERT Network

Finally, each member is asked to propose the best possible time and cost estimates for each of the activities with which they are most closely associated. Their estimates appear as follows (see Table I).

Table I
PERT Time and Cost Estimates

Activity	Predecessor Event	Successor Event	Normal Conditions				Crash Conditions	
			Optimistic Time (Weeks)	Most Likely Time (Weeks)	Pessimistic Time (Weeks)	Activity Cost $(00)	Activity Time (Weeks)	Activity Cost $(00)
A	1	2	4	5	12	140	3	500
B	1	3	3	3	3	90	3	90
C	2	3	2	4	6	160	2	360
D	3	4	3	4	11	110	3	410
E	2	5	1	2	3	80	1	120
F	4	5	2	4	6	100	2	260
G	5	6	1	3	5	130	1	410
H	4	6	2	5	8	120	2	390

Based on the above information and your knowledge of PERT Time and PERT Cost, you are asked to determine:

a)   Expected activity times (under normal conditions) -- t_e.

b)   Earliest completion time for each activity (under normal conditions) -- T_E.

c)   Latest allowable time for each activity (under normal conditions) -- T_L.

d)   Contractual completion date for project (under normal conditions) -- T_S.

e)   Activity slacks and critical path (under normal conditions).

f)   Probability of completing project on schedule (under normal conditions).

g)   Cost of reducing project completion time by one week.

Case Problem Answer

The answers to questions a) -- e) are shown in Table II and Figure II. It can be seen that the activities appearing on the critical path are A, C, D, F, G; each with a one-day slack. Activities B, E, and H carry slacks of 7, 11, and 2 weeks respectively.

The following represent the alternative activity paths by which the project is completed:

Path	Normal Time (weeks)
A, E, G	$6 + 2 + 3 = 11$
A, C, D, F, G	$6 + 4 + 5 + 4 + 3 = 22$
A, C, D, H	$6 + 4 + 5 + 5 = 20$
B, D, F, G	$3 + 5 + 4 + 3 = 15$
B, D, H	$3 + 5 + 5 = 13$

Path A, C, D, F, G is the critical path under normal conditions, and for a cost of $93,000 the project can be completed in 22 weeks. If the project is to be completed in 21 weeks activity along this critical path must be expedited under crash conditions, i.e., a one-week positive slack now becomes a one-week negative slack (slippage). The cost slope provides the answer as to which of the five activities along the critical path can be reduced by one week with the minimum increase in total project cost. Activity F has a $8000 cost slope (see Table III), i.e., activity F can be reduced in time from 4 to 3 weeks at a total project cost of $101,000 (total of $93,000 plus $8,000). This activity time reduction should allow the project to be completed within 21 weeks. The costs of further reductions in project completion times could be similarly determined.

Figure II
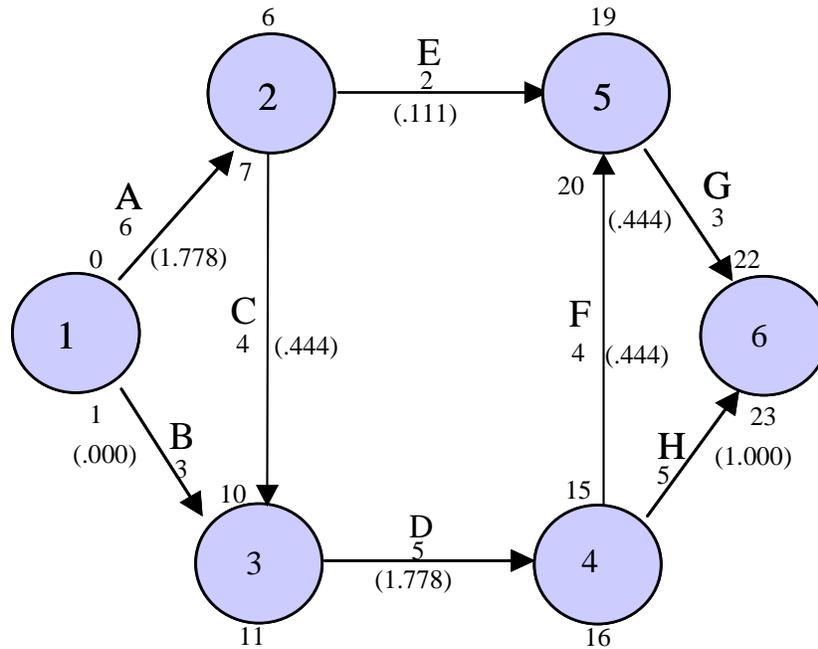
Critical Path = A, C, D,F, G

Table II
PERT Activity Output Slack Sort

Activity	Predecessor Event	Successor Event	Normal Conditions			t_e (wks)	σ^2	T_E	T_L	T_S	Slack	T_S-T_E	$\sum \sigma^2_{T_E}$	Z	Pr(%)
			a (wks)	m (wks)	b (wks)										
A	1	2	4	5	12	6	1.778	6	7		1				
B	1	3	3	3	3	3	.000	10	11		1				
C	2	3	2	4	6	4	.444	10	11		1				
D	3	4	3	4	11	5	1.778	15	16		1				
E	2	5	1	2	3	2	.111	19	20		1				
F	4	5	2	4	6	4	.444	19	20		1				
G	5	6	1	3	5	3	.444	22	23		1				
H	4	6	2	5	8	5	1.000	22	23	23	1	1	5.099	.1961	.974
							5.099								

a = optimistic time

m = most likely time

b = pessimistic time

t_e = expected time = $\dfrac{a + 4m + b}{6}$

σ^2 = estimate variation = $\left(\dfrac{b - a}{6}\right)^2$

T_E = earliest completion time

T_L = latest allowable time

T_S = contractual completion time

T_L-T_E = slack

Z = table Z value = $\dfrac{T_S - T_E}{\sqrt{\sum \sigma^2_{T_E}}}$

Pr = % probability of project completion on schedule

Table III
Time-Cost Tradeoff

Activity	Predecessor Event	Successor Event	Normal		Crash		Cost Slope
			t_e	Cost	Time	Cost	
			(weeks)	($00)	(weeks)	($00)	($00)
A	1	2	6	140	3	500	120
B	1	3	3	90	3	90	*
C	2	3	4	160	2	360	100
D	3	4	5	110	3	410	150
E	2	5	2	80	1	120	40
F	4	5	4	100	2	260	80
G	5	6	3	130	1	410	140
H	4	6	5	120	3	390	90
				$930		$2,540	

*This activity cannot be expedited.

$$\text{Cost Slope} = \frac{\text{Crash Cost - Normal Cost}}{\text{Normal Time - Crash Time}}$$